Available online at www.sciencedirect.com

SCIENCE DIRECT®

Chaos, Solitons and Fractals 23 (2005) 1013–1032

CHAOS SOLITONS & FRACTALS

www.elsevier.com/locate/chaos

Observer-based robust adaptive variable universe fuzzy control for chaotic system

Wang Jiang *, QiaoGuo-Dong , Deng Bin

School of Electrical and Automation Eng., Tianjin University, 300072 Tianjin, PR China

Accepted 8 June 2004

Abstract

A novel observer-base output feedback variable universe adaptive fuzzy controller is investigated in this paper. The contraction and expansion factor of variable universe fuzzy controller is on-line tuned and the accuracy of the system is improved. With the state-observer, a novel type of adaptive output feedback control is realized. A supervisory controller is used to force the states to be within the constraint sets. In order to attenuate the effect of both external disturbance and variable parameters on the tracking error and guarantee the states to be within the constraint sets, a robust controller is appended to the variable universe fuzzy controller. Thus, the robustness of system is improved. By *Lyapunov* method, the observer-controller system is shown to be stable. The overall adaptive control algorithm can guarantee the global stability of the resulting closed-loop system in the sense that all signals involved are uniformly bounded. In the paper, we apply the proposed control algorithms to control the *Duffing* chaotic system and *Chua's* chaotic circuit. Simulation results confirm that the control algorithm is feasible for practical application.

1. Introduction

Since fuzzy logic system is a universal approximator, the adaptive control [1–3] schemes of nonlinear system that incorporate the techniques of fuzzy logic are used to identify and control the nonlinear dynamic system [4–6]. According to *Universal Approximation Theorem* [7–11], for any given real continuous function $f(x)$ on a compact subset $U \subset R^N$ and arbitrary $\varepsilon > 0$, there exists a fuzzy system $y(x)$, such that $\max_{x \subset U} \|f(x) - g(x)\| < \varepsilon$. But there exists an error between the exact nonlinear system and an approximate model, which deteriorate the stability and control performance. Therefore, an adaptive fuzzy system, which can incorporate the expert information systematically, has been proposed to on-line tune fuzzy rules. Thus, the approximate error is decreased. The indirect adaptive fuzzy controller with observer [14–16] has been developed to control the unknown nonlinear dynamic system successfully. However, the direct adaptive fuzzy controller with observer [17], which has the advantage of no design effort to model the unknown plant, has seldom been shown. The design of fuzzy controller can be divided into two parts: fuzzy rules and scaling gains (contraction–expansion factors). Some articles [12,13,18–20] have shown the design approaches of fuzzy controller, but these approaches depend on operation experience. In this paper, a variable universe adaptive fuzzy controller with

* Corresponding author. Fax: +86 22 2740 9881.
E-mail address: jiangwang@tju.edu.cn (W. Jiang).

0960-0779/$ - see front matter
doi:10.1016/j.chaos.2004.06.020

observer is investigated to control a class of unknown nonlinear dynamical system. We know, different control values, namely different scaling gains (contraction–expansion factor) is needed in transient and steady states. Thus, by adjusting scaling gains (contraction–expansion factors), the universe of discourse is changed automatically.

There are at least two different approaches, which can guarantee the stability of fuzzy system. The first approach [12] is to specify the structure and parameters of fuzzy controller such that the closed-loop system is stable. But the approach usually requires fuzzy controller to satisfy some strong conditions, which greatly limit the design flexibility. In the second approach [7], the fuzzy controller is designed firstly without any stability consideration, and then another controller (supervisory controller) is appended to the fuzzy controller to satisfy the stability requirement. Because there is much flexibility in designing fuzzy controller in this second approach, the resulting system is expected to show high performance. Because variable universe fuzzy controller is the main controller, the supervisory controller would be a better safeguard. Therefore, the supervisory controller works in the following fashion: if the variable universe fuzzy controller works well, the supervisory controller is idle; if the fuzzy control system tends to be unstable, the supervisory controller begins to work in order to guarantee stability. Thus, all the signals involved are bounded.

The rest of this paper is organized as follows. In Section 2, the problem formulation is presented. Section 3, an observer-based variable universe fuzzy controller is developed. Cooperated with the supervisory controller and robust controller, scaling gains (contraction–expansion factors) are on-line tuned. Section 4, Simulation examples to demonstrate the performance of the proposed method are provided. Section 5, we make a conclusion of the advocated design methodology.

2. Problem formulation

Consider the nth-order nonlinear dynamical system of the form

$$\begin{aligned}
&\dot{x}_1 = x_2,\\
&\dot{x}_2 = x_3.\\
&\cdots\\
&\dot{x}_n = f(x_1x_2\cdots x_{n-1}x_n) + g(x_1x_2\cdots x_{n-1}x_n)u + d,\\
&y = x_1,
\end{aligned} \tag{1}$$

where, $f(\mathbf{x})$ and $g(\mathbf{x})$: unknown but bounded functions, $u\in R$ and $y\in R$: control input and output of the system, respectively. d: external bounded disturbance.

Define $\mathbf{x} = [x_1 \quad x_2 \quad \cdots \quad x_n] = [x \quad \dot{x} \quad \cdots \quad x^{(n-1)}] \in \mathfrak{R}^n$; the state space representation of (1) is expressed as

$$\begin{aligned}
&\dot{\mathbf{x}} = A + B(f(\mathbf{x}) + g(\mathbf{x})u + d),\\
&y = C^{\mathrm{T}}\mathbf{x},
\end{aligned} \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0\\ 0 & 0 & 1 & 0 & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0\\ 0\\ \vdots\\ 0\\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1\\ 0\\ 0\\ \vdots\\ 0 \end{bmatrix},$$

and $\mathbf{x}' = [x_2 \quad \cdots \quad x_n]^{\mathrm{T}} = [\dot{x} \quad \cdots \quad x^{(n-1)}]^{\mathrm{T}} \in \mathfrak{R}^{n-1}$ is a state vector where x_j $(j = 2,\ldots,n)$ are not assumed to be available for measurement. Only the output y is assumed to be measurable. For (2) to be controllable, it is required that $g(\mathbf{x}) \neq 0$ for $\mathbf{x}$ in a certain controllability region $U_c \subset \mathfrak{R}^n$. We assumed that $0 < g(\mathbf{x}) < \infty$, $\mathbf{x}\in U_c$. The control objective is to force the output y to follow a given bounded reference signal y_r. For the sake of facility, we transform a tracking problem into a regulation problem. The reference signal vector $\mathbf{y}_r$, tracking error vector $\mathbf{e}$ and estimation error vector $\hat{\mathbf{e}}$ are defined as, respectively,

$$\begin{aligned}
&\mathbf{y}_r = \begin{bmatrix} y_r & \dot{y}_r & \cdots & y_r^{(n-1)} \end{bmatrix}^{\mathrm{T}} \in \mathfrak{R}^n,\\
&\mathbf{e} = \mathbf{y}_r - \mathbf{x} = \begin{bmatrix} e & \dot{e} & \cdots & e^{(n-1)} \end{bmatrix}^{\mathrm{T}} \in \mathfrak{R}^n,\\
&\hat{\mathbf{e}} = \mathbf{y}_r - \hat{\mathbf{x}} = \begin{bmatrix} \hat{e} & \dot{\hat{e}} & \cdots & \hat{e}^{(n-1)} \end{bmatrix}^{\mathrm{T}} \in \mathfrak{R}^n,
\end{aligned} \tag{3}$$

where $\hat{\mathbf{x}}$ and $\hat{\mathbf{e}}$ denote the estimation of $\mathbf{x}$ and $\mathbf{e}$, respectively. Select $\mathbf{k}_c = [k_1^c \quad k_2^c \quad \cdots \quad k_n^c]^T \in \Re^n$ such that all roots of the polynomial $p(s) = s^n + k_n^c s^{n-1} + \cdots + k_1^c$ are in the open left-half plane, i.e. stable *Hurwitz* polynomial. If $f(\mathbf{x})$, $g(\mathbf{x})$ are known and the system is free of disturbances, based-on certainty equivalence approach, the control law is as follows:

$$u^* = \frac{1}{g(\mathbf{x})}\left[-f(\mathbf{x}) + y_r^{(n)} + \mathbf{k}_c^T \mathbf{e}\right]. \tag{4}$$

However, $f(\mathbf{x})$, $g(\mathbf{x})$ is unknown and not all states are available for measurement, we have to design an observer to estimate the state vector. We select a controller as follows:

$$u = u_D(\hat{\mathbf{x}}/\beta) + u_S(\hat{\mathbf{x}}) + u_C, \tag{5}$$

where $u_D(\hat{\mathbf{x}}/\beta)$ is the variable universe fuzzy controller; $u_S(\hat{\mathbf{x}})$; is the supervisory controller; u_C is the robust controller, and the certainty equivalent controller can be rewritten as

$$u^* = \frac{1}{\hat{g}(\hat{\mathbf{x}})}\left[-\hat{f}(\hat{\mathbf{x}}) + y_r^{(n)} + \mathbf{k}_c^T \hat{\mathbf{e}}\right]. \tag{6}$$

From (5), (6) and (2), we have

$$\begin{aligned}
\dot{\mathbf{e}} &= \dot{\mathbf{y}}_r - \dot{\mathbf{x}} = A\mathbf{y}_r + By_r^{(n)} - A\mathbf{x} - B\{f(\mathbf{x}) + g(\mathbf{x})[u_D(\hat{\mathbf{x}}/\beta) + u_S(\hat{\mathbf{x}}) + u_C] + d\} \\
&= A\mathbf{e} - B\mathbf{k}_c^T\hat{\mathbf{e}} - B\{-\hat{g}(\hat{\mathbf{x}})u^* + g(\mathbf{x})[u_D(\hat{\mathbf{x}}/\beta) + u_S(\hat{\mathbf{x}}) + u_C] + d\} \\
&= A\mathbf{e} - B\mathbf{k}_c^T\hat{\mathbf{e}} + B\{\hat{g}(\hat{\mathbf{x}})u^* - \hat{g}(\hat{\mathbf{x}})[u_D + u_S + u_C] + \hat{g}(\hat{\mathbf{x}})(u_D + u_S + u_C) - g(\mathbf{x})(u_D + u_S + u_C) - d\} \\
&= A\mathbf{e} - B\mathbf{k}_c^T\hat{\mathbf{e}} + B(\hat{g}(\hat{\mathbf{x}})(u^* - u_D - u_S - u_C) + (\hat{g}(\hat{\mathbf{x}}) - g(\mathbf{x}))(u_D + u_S + u_C) - d), \\
e_1 &= C^T\mathbf{e}.
\end{aligned} \tag{7}$$

Consider the following observer to estimate the error vector $\mathbf{e}$ in (7),

$$\begin{aligned}
\dot{\hat{\mathbf{e}}} &= A\hat{\mathbf{e}} - B\mathbf{k}_c^T\hat{\mathbf{e}} + \mathbf{k}(e_1 - \hat{e}_1), \\
\hat{e}_1 &= C^T\hat{\mathbf{e}}.
\end{aligned} \tag{8}$$

The observable errors are defined as $\tilde{\mathbf{e}} = \mathbf{e} - \hat{\mathbf{e}}$, from (7) and (8), we can obtain observable error vector as follows:

$$\begin{aligned}
\dot{\tilde{\mathbf{e}}} &= (A - \mathbf{k}C^T)\tilde{\mathbf{e}} + B[\hat{g}(\hat{\mathbf{x}})(u^* - u_D(\hat{\mathbf{x}}/\beta) - u_S - u_C) + (\hat{g}(\hat{\mathbf{x}}) - g(\mathbf{x}))(u_D + u_S + u_C) - d], \\
\tilde{e}_1 &= C^T\tilde{\mathbf{e}},
\end{aligned} \tag{9}$$

where

$$A - \mathbf{k}C^T = \begin{bmatrix} -k_n & 1 & 0 & 0 & \cdots & 0 & 0 \\ -k_{n-1} & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -k_2 & 0 & 0 & 0 & 0 & 0 & 1 \\ -k_1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \mathbf{k} = \begin{bmatrix} k_n \\ k_{n-1} \\ \vdots \\ k_2 \\ k_1 \end{bmatrix}, \tag{10a}$$

$$(A - \mathbf{k}C^T)^T P + P(a - \mathbf{k}C^T) = -Q, \quad Q \text{ is arbitrary positive definite matrix.} \tag{10b}$$

Since $(C, A - \mathbf{k}C^T)$ pair is observable, select the observer gain vector $\mathbf{k}$, such that the characteristic of polynomial $A - \mathbf{k}C^T$ is strictly Hurwitz [6]. Thus, there exits a positive definite symmetric matrix P, which satisfies the Lyapunov equation (10b).

3. Variable universe adaptive fuzzy controller

3.1. Basic structures [21,22]

Let $X_j = [-E_j, E_j]$ $(j = 1,2,\ldots,n)$ be the universe discourse of the input variable x_j $(j = 1,2,\ldots,n)$, and $Y = [-U, U]$ be the universe discourse of the output variable y. $\{A_{jl}\}_{(1\leqslant l\leqslant h)}$ stand for a fuzzy partition on X_j and $\{B_l\}_{(1\leqslant l\leqslant h)}$ stand for a fuzzy partition on Y. For any $x_j \in X_j$, the membership $A_{jl}(x_j)$ which is the true value of "x_j is A_{jl}" is transferred to the resulting consequent parameter y_l. If the value is 1, the resulting value of the consequent parameter is certainly y_l.

However, $A_{jl}(x_j)$ is not always equal to 1. So, y_l is not completely chosen as the resulting consequent parameter value. We choose a "reliability" which is not higher than $A_{jl}(x_j)$ to be a weight multiplied by y_l. In the paper, the "reliability" is equal to $A_{jl}(x_j)$, the resulting output is as follows:

$$y \triangleq \frac{\left(\sum_{l=1}^{h}\prod_{j=1}^{n} A_{jl}(x_j)y_l\right)}{\sum_{l=1}^{h} A_{jl}(x_j)}, \quad \sum_{l=1}^{h} A_{jl}(x_j) = 1. \tag{11}$$

In the paper, the variable universe fuzzy controller is presented. In the premise that the number of initial control rules is fixed, the universe discourse is changed with the changing error. Thus, the control rules are tuned dynamically. The situation of variable universe is shown in Fig. 1.

The transformed universe discourse is denoted as

$$X_j(x_j) = [-\alpha_j(x_j)E_j \quad \alpha_j(x_l)E_j], \qquad Y(y) = [-\beta(y)U \quad \beta(y)U],$$

where $\alpha_j(x_j)$, $\beta(y)$ are contraction–expansion factors. Generally speaking, we give the following contraction–expansion factor (for detailed reasoning, refer to [21–24]); $\alpha(x) = 1 - \lambda\exp(-\kappa x^2)$, $\lambda \in (0,1)$, $k > 0$. The output of the variable universe fuzzy controller is represented as

$$u_C(\hat{\mathbf{x}}/\beta) = \beta \sum_{l=1}^{h}\prod_{j=1}^{n} A_{jl}\left(\frac{\hat{x}_j}{\hat{\alpha}(\hat{x}_j)}\right) y_l. \tag{12}$$

From (12), we can select a reasonable β to optimize the adaptive laws.

3.2. The essence of variable universe fuzzy controller

The essence of variable universe fuzzy controller is an improved *PD* fuzzy controller. A conventional *PD* fuzzy control algorithm is

$$u(k) = C_e e(k) + C_{\Delta e} e(k), \tag{13}$$

where $e(k)$ and $\Delta e(k)$ are the values of error and the change of error at the kth sample time, respectively. If $e(k)$ and $\Delta e(k)$ are fuzzy variable, (13) becomes a fuzzy control algorithm:

$$u(k) = C_U * F(E, \Delta E) = C_U * F[C_e e(k), C_{\Delta e}\Delta e(k)], \tag{14}$$

where $e(k)$ = desired value y_r-output value y (at the kth sampling time), and $\Delta e(k) = e(k) - e(k-1)$.

3.3. Directive adaptive variable universe fuzzy controller

An adaptive fuzzy controller that uses fuzzy logic system as a model of the unknown plant is an indirect fuzzy controller, which can incorporate fuzzy description of the unknown plant, but cannot incorporate the control rules. On the other hand, an adaptive fuzzy controller that directly uses fuzzy logic system as a controller is a direct fuzzy controller, which can incorporate control rules, but cannot incorporate fuzzy description of the unknown plant. In the paper, we develop a direct adaptive fuzzy control.

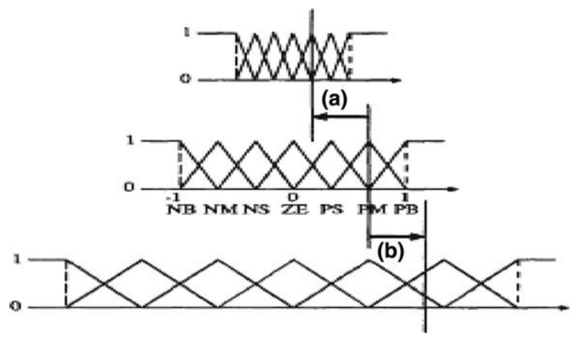

Fig. 1. The change of universe discourse.

Considering the error dynamical equation (9), select Lyapunov function

$$V_{\tilde{e}} = \frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}} P\tilde{\mathbf{e}}. \tag{15}$$

Differentiating (15) with respect to time, we have,

$$\begin{aligned}\dot{V}_{\tilde{\mathbf{e}}} &= \frac{1}{2}\dot{\tilde{\mathbf{e}}}^{\mathrm{T}} P\tilde{\mathbf{e}} + \frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}} P\dot{\tilde{\mathbf{e}}}\\ &= -\frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}} Q\tilde{\mathbf{e}} + \tilde{\mathbf{e}}^{\mathrm{T}} PB\{\hat{g}(\hat{\mathbf{x}})(u^* - u_{\mathrm{D}} - u_{\mathrm{S}} - u_{\mathrm{C}}) + [\hat{g}(\hat{\mathbf{x}}) - g(\mathbf{x})](u_{\mathrm{D}} + u_{\mathrm{S}} + u_{\mathrm{C}}) - d\}\\ &\leqslant -\frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}} Q\tilde{\mathbf{e}} + |\tilde{\mathbf{e}}^{\mathrm{T}} PB|(|\hat{g}(\hat{\mathbf{x}})u^*| + |g(\mathbf{x})(u_{\mathrm{D}} + u_{\mathrm{C}})| + |d|) - \tilde{\mathbf{e}}^{\mathrm{T}} PBg(\mathbf{x})u_{\mathrm{S}}\\ &\leqslant -\frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}} Q\tilde{\mathbf{e}} + |\tilde{\mathbf{e}}^{\mathrm{T}} PB|(|f(\mathbf{x})| + |y_{\mathrm{r}}^{(n)}| + |\mathbf{k}_{\mathrm{c}}^{\mathrm{T}}\hat{\mathbf{e}}| + |g(\mathbf{x})(u_{\mathrm{D}} + u_{\mathrm{C}})| + |d|) - \tilde{\mathbf{e}}^{\mathrm{T}} PBg(\mathbf{x})u_{\mathrm{S}}.\end{aligned} \tag{16a}$$

In order to design u_{S}, such that $\dot{V}_{\tilde{\mathbf{e}}} \leqslant 0$. We can choose $\mathbf{k}$ to guarantee $\mathbf{x} \approx \hat{\mathbf{x}}$. The following constraint conditions are needed. Let $f^{\mathrm{U}}(\mathbf{x})$, $g_{\mathrm{L}}(\mathbf{x})$, $g_{\mathrm{U}}(\mathbf{x})$ satisfy

$$\begin{aligned}&|f(\mathbf{x})| \leqslant f^{\mathrm{U}}(\mathbf{x}) \approx f^{\mathrm{U}}(\hat{\mathbf{x}}) < \infty,\\ &0 < g_{\mathrm{L}}(\hat{\mathbf{x}}) \approx g_{\mathrm{L}}(\mathbf{x}) \leqslant g(\mathbf{x}) \leqslant g_{\mathrm{U}}(\mathbf{x}) \approx g_{\mathrm{U}}(\hat{\mathbf{x}}) < \infty,\\ &\mathbf{x} \in U_{\mathrm{C}}, \quad |d| \leqslant D_{\mathrm{N}}.\end{aligned} \tag{16b}$$

Observe (16a), using (16b), the supervisory control u_{S} is chosen as

$$u_{\mathrm{S}} = I^* \operatorname{sgn}(\tilde{\mathbf{e}}^{\mathrm{T}} PB)\frac{1}{g_{\mathrm{L}}(\mathbf{x})}\left(\left|g^{\mathrm{U}}(\hat{\mathbf{x}})(u_{\mathrm{D}} + u_{\mathrm{C}})\right| + \frac{g_{\mathrm{U}}(\mathbf{x})}{g_{\mathrm{L}}(\mathbf{x})}\left(f_{\mathrm{U}}(\mathbf{x}) + |y_{\mathrm{r}}^{n}| + |\mathbf{k}_{\mathrm{c}}^{\mathrm{T}}\hat{\mathbf{e}}|\right) + D_{\mathrm{N}}\right), \tag{17}$$

where,

$$I^* = \begin{cases} 1 & V_{\tilde{e}} > \overline{V},\\ 0 & V_{\tilde{e}} \leqslant \overline{V}, \end{cases} \tag{18}$$

$\overline{V}$ is a constant, which is chosen by the designer. Considering the case $V_{\tilde{e}} > \overline{V}$, substituting (4) and (17) into (16), we can get

$$\begin{aligned}\dot{V}_{\tilde{e}} \leqslant &-\frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}} Q\tilde{\mathbf{e}} + |\tilde{\mathbf{e}}^{\mathrm{T}} PB|\left(|f(\mathbf{x})| + |y_{\mathrm{r}}^{(n)}| + |\mathbf{k}_{\mathrm{c}}^{\mathrm{T}}\hat{\mathbf{e}}| + |g(\mathbf{x})(u_{\mathrm{D}} + u_{\mathrm{C}})| + |d|\right)\\ &- |\tilde{\mathbf{e}}^{\mathrm{T}} PB|\frac{g(\mathbf{x})}{g_{\mathrm{L}}(\mathbf{x})}\left(|g_{\mathrm{U}}(\mathbf{x})(u_{\mathrm{D}} + u_{\mathrm{C}})| + \frac{g_{\mathrm{U}}(\mathbf{x})}{g_{\mathrm{L}}(\mathbf{x})}(f_{\mathrm{U}}(\hat{\mathbf{x}}) + |y_{\mathrm{r}}^{n}| + |\mathbf{k}_{\mathrm{c}}^{\mathrm{T}}\tilde{\mathbf{e}}|) + D_{\mathrm{N}}\right)\\ = &-\frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}} Q\tilde{\mathbf{e}} + |\tilde{\mathbf{e}}^{\mathrm{T}} PB| * \Bigg(|f(\mathbf{x})| + |y_{\mathrm{r}}^{n}| + |\mathbf{k}_{\mathrm{c}}^{\mathrm{T}}\hat{\mathbf{e}}| + |g(\mathbf{x})(u_{\mathrm{D}} + u_{\mathrm{C}})| + |d|.\\ &- \frac{g(\mathbf{x})}{g_{\mathrm{L}}(\mathbf{x})}\left(|g_{\mathrm{U}}(\mathbf{x})(u_{\mathrm{D}} + u_{\mathrm{C}})| + \frac{g_{\mathrm{U}}(\mathbf{x})}{g_{\mathrm{L}}(\mathbf{x})}\left(f_{\mathrm{U}}(\mathbf{x}) + |y_{\mathrm{r}}^{n}| + |\mathbf{k}_{\mathrm{c}}^{\mathrm{T}}\tilde{\mathbf{e}}|\right) + D_{\mathrm{N}}\right)\Bigg) \leqslant -\frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}} Q\tilde{\mathbf{e}}.\end{aligned} \tag{19}$$

Since u_{S} plays the roles of "rough regulation", we can always guarantee $V_{\tilde{e}} \leqslant \overline{V}$. Because $\mathbf{P}$ is positive definite symmetry matrix, $V_{\tilde{e}} \leqslant \overline{V}$ implies the bounded of $\tilde{\mathbf{e}}$, which in turn implies the bounds of $\hat{\mathbf{e}}$ and $\hat{\mathbf{x}}$. It is obvious that the supervisory controller is nonzero when $V_{\tilde{e}}$ is greater than $\overline{V}$. Thus, if the closed-loop system with fuzzy controller (5) is stable, i.e. the error is small, and then the supervisory controller is idle. On the other hand, if the system tends to diverse, then the supervisory controller begins to operate to force $V_{\tilde{e}} \leqslant \overline{V}$.

In order to adjust the parameters in the fuzzy system, we should derive the adaptive law. Hence, the optimal parameters are defined as following:

$$\begin{aligned}\beta^* &= \underset{|\beta| \leqslant N_\beta}{\operatorname{argmin}}\left(\left|\sup_{|\hat{x}| \leqslant N_{\hat{x}}}(u^* - u_{\mathrm{D}}(\hat{\mathbf{x}}/\beta))\right|\right),\\ \alpha^* &= \underset{|\alpha| \leqslant N_\alpha}{\operatorname{argmin}}\left(\left|\sup_{|\hat{x}| \leqslant N_x}(g(\mathbf{x}) - \hat{g}(\hat{\mathbf{x}}/\alpha))\right|\right),\end{aligned} \tag{20}$$

where N_β and N_α are compact sets of suitable bounds of α and β, respectively. Define the minimum approximate error as

$$\theta = -\hat{g}(\hat{\mathbf{x}}/\alpha)(u^* - u_{\mathrm{D}}(\hat{\mathbf{x}}/\beta^*)) + (u_{\mathrm{D}} + u_{\mathrm{C}})(g(\mathbf{x}) - \hat{g}(\hat{\mathbf{x}}/\alpha^*)) + d. \tag{21}$$

The error dynamics (9) can be expressed as

$$\dot{\tilde{\mathbf{e}}} = (A - \mathbf{k}C^{\mathrm{T}})\tilde{\mathbf{e}} + B(\hat{g}(\hat{\mathbf{x}}/\alpha)(u_{\mathrm{D}}(\hat{\mathbf{x}}/\beta^{*}) - u_{\mathrm{D}}(\hat{\mathbf{x}}/\beta) - u_{\mathrm{C}}) - (u_{\mathrm{D}} + u_{\mathrm{C}})(\hat{g}(\hat{\mathbf{x}}/\alpha^{*}) - \hat{g}(\hat{\mathbf{x}}/\alpha))) - B\theta - Bg(\mathbf{x})u_{\mathrm{S}}. \tag{22}$$

In order to derive the parameter adaptive laws to on-line tune the parameters α and β, we need to use the variable universe adaptive fuzzy controller to approximate $u_{\mathrm{D}}(\hat{\mathbf{x}}/\beta)$, $g(\hat{\mathbf{x}}/\alpha)$. Thus, we can obtain

$$\begin{aligned}
&u_{\mathrm{D}}(\hat{\mathbf{x}}/\beta) = \beta\varepsilon(\hat{\mathbf{x}}), \qquad \varepsilon(\hat{\mathbf{x}}) = \sum_{l=1}^{h}\prod_{j=1}^{n} A_{jl}\left(\frac{x_j}{\alpha(x_j)}\right) * y_l,\\
&\hat{g}(\hat{\mathbf{x}}/\alpha) = \alpha\eta(\hat{\mathbf{x}}), \qquad \eta(\hat{\mathbf{x}}) = \sum_{l=1}^{h}\prod_{j=1}^{n} A_{jl}\left(\frac{x_j}{\alpha(x_j)}\right) * y_l,\\
&u_{\mathrm{C}} = \frac{\sup_{t\geqslant 0}|\theta|}{g_{\mathrm{L}}(x)}\operatorname{sgn}(e^{\mathrm{T}}PB),
\end{aligned} \tag{23}$$

where u_{C} is used to attenuate the effect of the approximate error and external disturbance on the tracking error. Thus, the dynamics (22) can be expressed as follows:

$$\dot{\tilde{\mathbf{e}}} = (A - \mathbf{k}c^{\mathrm{T}})\tilde{\mathbf{e}} + B(\hat{g}(\hat{\mathbf{x}}/\alpha)\tilde{\beta}\varepsilon(\hat{\mathbf{x}}) - u_{\mathrm{D}}(\hat{\mathbf{x}}/\beta)\tilde{\alpha}\eta(\hat{\mathbf{x}}) - \hat{g}(x/\alpha^{*})u_{\mathrm{C}}) - B\theta - Bg(\mathbf{x})u_{\mathrm{S}}. \tag{24}$$

Define the following variables

$$\tilde{\beta} = \beta^{*} - \beta, \qquad \tilde{\alpha} = \alpha^{*} - \alpha, \tag{25}$$

where $\varepsilon(\hat{\mathbf{x}})$ and $\eta(\hat{\mathbf{x}})$ are fuzzy basic elements.

In order to derive adaptive laws, we consider the following Lyapunov function:

$$V = \frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}}P\tilde{\mathbf{e}} + \frac{1}{2\chi}\tilde{\beta}^{2} + \frac{1}{2\kappa}\tilde{\alpha}^{2}. \tag{26}$$

Differentiate (26) with respect to time along the trajectory (24), then

$$\begin{aligned}
\dot{V} &= \frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}}P\dot{\tilde{\mathbf{e}}} + \frac{1}{2}\dot{\tilde{\mathbf{e}}}^{\mathrm{T}}P\tilde{\mathbf{e}} + \frac{\tilde{\beta}\dot{\tilde{\beta}}}{\chi} + \frac{\dot{\tilde{\alpha}}\tilde{\alpha}}{\kappa}\\
&= \frac{1}{2}\left\{(A - \mathbf{k}c^{\mathrm{T}})\tilde{\mathbf{e}} + B[\hat{g}(\hat{\mathbf{x}}/\alpha)\tilde{\beta}\varepsilon(\hat{\mathbf{x}}) - u_{\mathrm{D}}(\hat{\mathbf{x}}/\beta)\tilde{\alpha}\eta(\hat{\mathbf{x}}) - \hat{g}(\mathbf{x}/\alpha^{*})u_{\mathrm{C}}] - B\theta - Bg(\mathbf{x})u_{\mathrm{S}}\right\}^{\mathrm{T}}P\tilde{\mathbf{e}}\\
&\quad + \frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}}P\left\{(A - \mathbf{k}c^{\mathrm{T}})\tilde{\mathbf{e}} + B[\hat{g}(\hat{\mathbf{x}}/\alpha)\tilde{\beta}\varepsilon(\hat{\mathbf{x}}) - u_{\mathrm{D}}(\hat{\mathbf{x}}/\beta)\tilde{\alpha}\eta(\hat{\mathbf{x}}) - \hat{g}(\mathbf{x}/\alpha^{*})u_{\mathrm{C}}] - B\theta - Bg(\mathbf{x})u_{\mathrm{S}}\right\} + \frac{\tilde{\beta}\dot{\tilde{\beta}}}{\chi} + \frac{\tilde{\alpha}\dot{\tilde{\alpha}}}{\kappa}\\
&= -\frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}}Q\tilde{\mathbf{e}} + \frac{\tilde{\beta}}{\chi}[\chi\tilde{\mathbf{e}}^{\mathrm{T}}PB\hat{g}(\hat{\mathbf{x}}/\alpha)\varepsilon(\hat{\mathbf{x}}) + \dot{\tilde{\beta}}] + \frac{\tilde{\alpha}}{\kappa}[\dot{\tilde{\alpha}} - \kappa\tilde{\mathbf{e}}^{\mathrm{T}}PBu_{\mathrm{D}}(\hat{\mathbf{x}}/\beta)\eta(\hat{\mathbf{x}})] - \tilde{e}^{\mathrm{T}}PB\theta - \tilde{e}^{\mathrm{T}}PBg(\mathbf{x})u_{\mathrm{S}}\\
&\quad - \tilde{\mathbf{e}}^{\mathrm{T}}PB\hat{g}(\mathbf{x}/\alpha^{*})u_{\mathrm{C}}.
\end{aligned} \tag{27}$$

Form the definition of u_{S}; we know $\tilde{e}^{\mathrm{T}}PBg(\mathbf{x})u_{\mathrm{S}} > 0$, if we choose the adaptive law as

$$\dot{\tilde{\beta}} = -\chi\tilde{e}^{\mathrm{T}}PB\hat{g}(\hat{\mathbf{x}}/\alpha)\varepsilon(\hat{\mathbf{x}}), \qquad \dot{\tilde{\alpha}} = \kappa\tilde{e}^{\mathrm{T}}PBu_{\mathrm{D}}(\hat{\mathbf{x}}/\beta)\eta(\hat{\mathbf{x}}). \tag{28}$$

Then, dynamics (27) can be expressed as,

$$\dot{V} \leqslant -\frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}}Q\tilde{\mathbf{e}} - \tilde{\mathbf{e}}^{\mathrm{T}}PB\theta - \tilde{e}^{\mathrm{T}}PB\hat{g}(x/\alpha^{*})u_{\mathrm{C}}. \tag{29}$$

From (23), we can get

$$\dot{V} \leqslant -\frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}}Q\tilde{\mathbf{e}} \leqslant 0, \tag{30}$$

we know that $\alpha^{*}\in N_{\alpha}$, $\beta^{*}\in N_{\beta}$. Thus, if we can constrain α, β within the sets N_{α}, N_{β}, then the $u_{\mathrm{D}}(\hat{\mathbf{x}}/\beta)$ will be bounded. From (17), we know that u_{S} will be bounded, and it should be reminded that $\tilde{e}$ would be bounded because of the supervisory controller. Obviously, the adaptive law (28) cannot guarantee that $\alpha\in N_{\alpha}$, $\beta\in N_{\beta}$, therefore all the adaptive laws should be modified using the parameter projection algorithms, such that all the parameters will remain inside the constraint sets. The modified algorithm is shown as following:

$$\begin{aligned}
&\Pr oj[\chi\tilde{\mathbf{e}}^{\mathrm{T}}PB\hat{g}(\hat{\mathbf{x}}/\alpha)\varepsilon(\hat{\mathbf{x}})] = \chi\tilde{\mathbf{e}}^{\mathrm{T}}PB\hat{g}(\hat{\mathbf{x}}/\alpha)\varepsilon(\hat{\mathbf{x}}) - \chi\tilde{\mathbf{e}}^{\mathrm{T}}PB\hat{g}(\hat{\mathbf{x}}/\alpha)\frac{\alpha\alpha^{\mathrm{T}}}{|\alpha|^2}\varepsilon(\hat{\mathbf{x}}),\\
&\Pr oj[\kappa\tilde{\mathbf{e}}^{\mathrm{T}}PBu_{\mathrm{D}}(\hat{\mathbf{x}}/\beta)\eta(\hat{\mathbf{x}})] = \kappa\tilde{\mathbf{e}}^{\mathrm{T}}PBu_{\mathrm{D}}(\hat{\mathbf{x}}/\beta)\eta(\hat{\mathbf{x}}) - \kappa\tilde{\mathbf{e}}^{\mathrm{T}}PBu_{\mathrm{D}}(\hat{\mathbf{x}}/\beta)\frac{\beta\beta^{\mathrm{T}}}{|\beta|^2}\eta(\hat{\mathbf{x}}).
\end{aligned} \tag{31}$$

Using the projection algorithm to tune the parameter vector α, β, we can get:

$$\dot{\beta} = \begin{cases} \chi\tilde{\mathbf{e}}^{\mathrm{T}}PB\hat{g}(\hat{\mathbf{x}}/\alpha)\varepsilon(\hat{\mathbf{x}}) & \text{if } |\beta| \leqslant N_\beta \ \text{(or } |\beta| = N_\beta \text{ and } \tilde{\mathbf{e}}^{\mathrm{T}}PB\hat{g}(\hat{\mathbf{x}}/\alpha)\alpha^{\mathrm{T}}\varepsilon(\hat{\mathbf{x}}) \leqslant 0),\\ \Pr oj(\chi\tilde{\mathbf{e}}^{\mathrm{T}}PB\hat{g}(\hat{\mathbf{x}}/\alpha)\varepsilon(\hat{\mathbf{x}})) & \text{if } |\beta| = N_\beta \text{ and } \tilde{\mathbf{e}}^{\mathrm{T}}PB\hat{g}(\hat{\mathbf{x}}/\alpha)\alpha^{\mathrm{T}}\varepsilon(\hat{\mathbf{x}}) \geqslant 0, \end{cases} \tag{32}$$

$$\dot{\alpha} = \begin{cases} \kappa\tilde{\mathbf{e}}^{\mathrm{T}}PBu_{\mathrm{D}}(\hat{\mathbf{x}}/\beta)\eta(\hat{\mathbf{x}}) & \text{if } |\alpha| \leqslant N_\alpha \ \text{(or } |\alpha| = N_\alpha \text{ and } \tilde{\mathbf{e}}^{\mathrm{T}}PBu_{\mathrm{D}}(\hat{\mathbf{x}}/\beta)\beta^{\mathrm{T}}\eta(\hat{\mathbf{x}}) \leqslant 0),\\ \Pr oj(\kappa\tilde{\mathbf{e}}^{\mathrm{T}}PBu_{\mathrm{D}}(\hat{\mathbf{x}}/\beta)\eta(\hat{\mathbf{x}})) & \text{if } |\alpha| = N_\alpha \text{ and } \tilde{\mathbf{e}}^{\mathrm{T}}PBu_{\mathrm{D}}(\hat{\mathbf{x}}/\beta)\beta^{\mathrm{T}}\eta(\hat{\mathbf{x}}) \geqslant 0. \end{cases} \tag{33}$$

Following the preceding, we have the following theorem:

Theorem. *Consider the plant (2) with the control law (5), where $u_{\mathrm{C}}(\hat{\mathbf{x}}/\beta)$ is given by (12) and u_S is given by (17). The parameter vector α, β is on-line tuned by the adaptive (32) and (33). Then, the parameters α, β and observe state vector $\hat{\mathbf{x}}$ of the plant (2) will be globally bounded, i.e. satisfying the following conditions*:

$$\begin{aligned}
&|\beta| \leqslant N_\beta, \qquad |\alpha| \leqslant N_\alpha,\\
&|\hat{\mathbf{x}}| \leqslant |y_{\mathrm{r}}| + \left(\frac{2\overline{V}_{\tilde{e}}}{\lambda_{\hat{P}\min}}\right)^{\frac{1}{2}} = N_{\hat{\mathbf{x}}}.
\end{aligned} \tag{34}$$

Moreover, using the corollary of Barbalet's *lemma, we can get* $\lim_{t\to\infty}|\tilde{\mathbf{e}}(t)| = 0$.

Proof. Select the Lyapunov function $V_\beta = \frac{1}{2}\beta^{\mathrm{T}}\beta$, if the first line of (32) is true, then:

(i) $|\beta| \leqslant N_\beta$, and $\dot{V}_\beta = \chi\tilde{\mathbf{e}}^{\mathrm{T}}PB\hat{g}(\hat{\mathbf{x}}/\alpha)\alpha^{\mathrm{T}}\varepsilon(\hat{\mathbf{x}}) \leqslant 0$. Thus, $\dot{\beta} \leqslant 0$.
(ii) $|\beta| = N_\beta$, and $\dot{V}_\beta = \chi\tilde{\mathbf{e}}^{\mathrm{T}}PB\hat{g}(\hat{\mathbf{x}}/\alpha)\alpha^{\mathrm{T}}\varepsilon(\hat{\mathbf{x}}) \leqslant 0$. Thus, $\dot{\beta} \leqslant 0$.

If the second line of (32) is true, i.e. $|\beta| = N_\beta$. Differentiate $V_\beta = \frac{1}{2}\beta^{\mathrm{T}}\beta$ with respect to time. Then, we can get

$$\dot{V}_\beta = \tilde{\mathbf{e}}^{\mathrm{T}}PB\beta^{\mathrm{T}}\hat{g}(\hat{\mathbf{x}}/\beta)\varepsilon(\hat{\mathbf{x}}) - \tilde{\mathbf{e}}^{\mathrm{T}}PB\hat{g}(\hat{\mathbf{x}}/\beta)\frac{|\beta|^2\beta^{\mathrm{T}}\varepsilon(\hat{\mathbf{x}})}{|\beta|^2} = 0. \tag{35}$$

So, we can get $|\beta| \leqslant N_\beta$. Using the same method, we can get

$$|\alpha| \leqslant N_\alpha. \tag{36}$$

Because of $V_{\hat{e}} \leqslant \overline{V}_{\hat{e}}$, i.e. $\frac{1}{2}\lambda_{P\min}|\hat{\mathbf{e}}|^2 \leqslant \frac{1}{2}\hat{e}^{\mathrm{T}}P\hat{\mathbf{e}} \leqslant \overline{V}_{\hat{e}}$, where $\lambda_{P\min}$ is the minimum eigenvalue of the matrix $\mathbf{P}$. After simple manipulation, we can obtain $|\hat{\mathbf{e}}| \leqslant \left(\frac{2\overline{V}}{\lambda_{P\min}}\right)^{\frac{1}{2}}$. Together with $\hat{\mathbf{e}} = \mathbf{y}_{\mathrm{r}} - \hat{\mathbf{x}}$, we have

$$|\hat{\mathbf{x}}| \leqslant \left[|\mathbf{y}_{\mathrm{r}}| + \left(\frac{2\overline{V}}{\lambda_{P\min}}\right)^{\frac{1}{2}}\right] = N_{\hat{x}}. \tag{37}$$

From (30), we can obtain

$$\dot{V} \leqslant -\tilde{\mathbf{e}}^{\mathrm{T}}Q\tilde{\mathbf{e}} \leqslant -\frac{\lambda_{Q\min}}{2}|\tilde{\mathbf{e}}|^2, \tag{38}$$

where $\lambda_{Q\min}$ is the minimum eigenvalue of the matrix Q. Integrating both sides of (38), after simple manipulations, we can obtain

$$\int_0^t |\tilde{\mathbf{e}}(\zeta)|^2 \mathrm{d}\varsigma \leqslant \frac{2}{\lambda_{Q\min-1}}(|V(0)| + |V(t)|). \tag{39}$$

From (39), we know, $\tilde{\mathbf{e}} \in \mathbf{L}_2$, $\tilde{\mathbf{e}} \in \mathbf{L}_\infty$. Since all the variables will be bounded in (22), then $\dot{\tilde{\mathbf{e}}} \in \mathbf{L}_\infty$. By the Barbalat's theorem,

$$\lim_{t\to\infty}(|\tilde{\mathbf{e}}(t)|) = 0. \qquad \square \tag{40}$$

4. Example

In the section, we will apply the proposed control algorithm to control the Duffing chaotic system and Chua's chaotic circuit to track a sine-wave trajectory.

Example 1. Consider the Duffing chaotic system whose dynamics is as following:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -0.1x_2 - x_1^3 + 12\cos t + u(t) + d, \end{cases} \\ y = [1 \quad 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{41}$$

where $u(t)$ is control input; d is bounded external disturbance. If $u(t) = 0$, then the system is chaotic system. The trajectories of the states x_1 and x_2 are shown in Figs. 2 and 3, respectively. The phase plane is shown in Fig. 4.

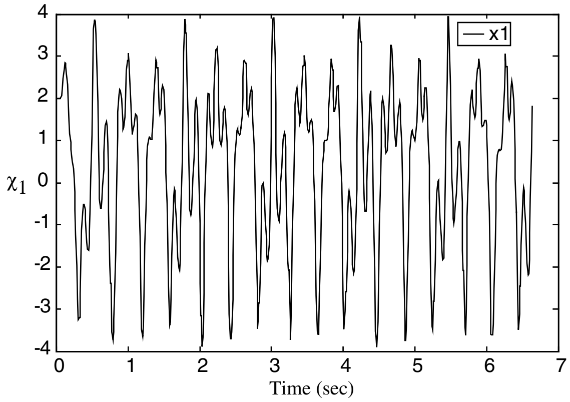

Fig. 2. The trajectory of x_1 without controller.

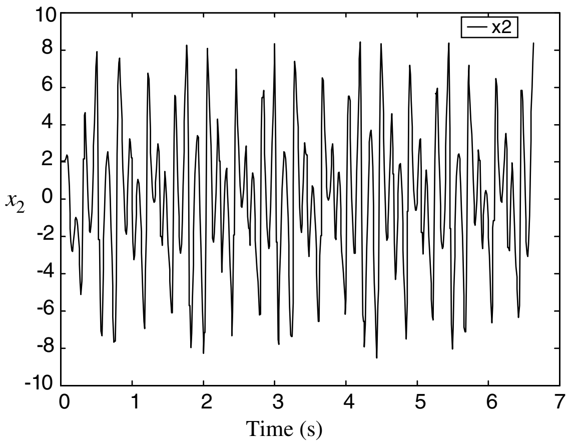

Fig. 3. The trajectory of x_2 without control.

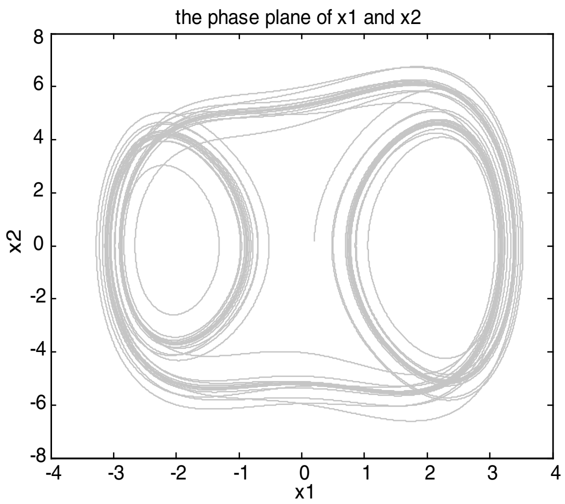

Fig. 4. Phase plane without control.

Using the variable universe fuzzy controller via output feedback to control Duffing chaotic system in order to force the states x_1 and x_2 to track the given bounded reference signals $y_r(t) = \sin(t)$ and $\dot{y}_r(t) = \cos(t)$. If the supervisory controller and robust controller are not appended to the fuzzy controller, the observable results and tracking results are not good. The trajectories of x_1, $\hat{x}_1$ and x_2, $\hat{x}_2$ are shown in Figs. 5 and 6, respectively. The desired output y_r and the actual output y are shown in Fig. 7. From these figures, we can see that there exists error between the actual states and the observable states. Moreover, the desired output is not tracked by the actual output completely. Thus, the supervisory controller and robust controller need be appended to the variable universe fuzzy controller in order to control the Duffing chaotic system. The simulation results are shown as follows. In order to satisfy the constraint conditions in design, define the following functions:

$$\begin{aligned} &f^{\mathrm{U}}(x_1,x_2) = 13 + |x_1^3| \approx 13 + |\hat{x}_1^3| = f^{\mathrm{U}}(\hat{x}_1,\hat{x}_2),\\ &g^{\mathrm{U}}(x_1,x_2) \approx g^{\mathrm{U}}(\hat{x}_1,\hat{x}_2) = 1.03,\\ &g_{\mathrm{L}}(x_1,x_2) \approx g_{\mathrm{L}}(\hat{x}_1,\hat{x}_2) = 0.59. \end{aligned} \tag{42}$$

Let the external disturbance d be a step signal. The membership functions of the error and the error change are shown in Figs. 8 and 9.

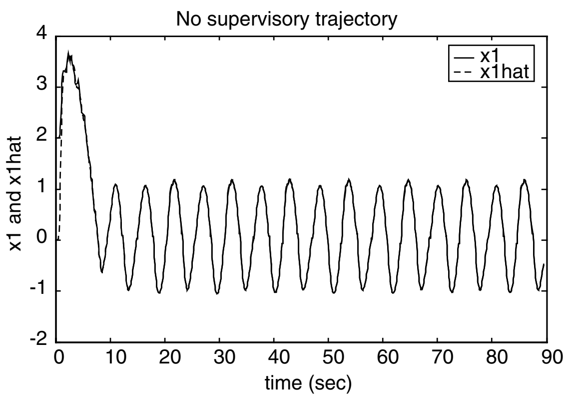

Fig. 5. The trajectory of x_1 and $\hat{x}_1$ only with $u_C(\hat{\mathbf{x}}/\beta)$.

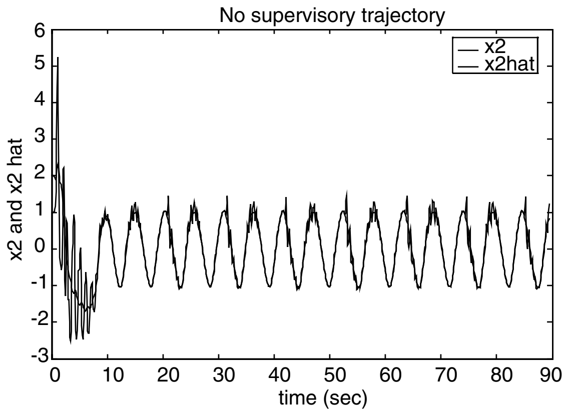

Fig. 6. The trajectory of x_2 and $\hat{x}_2$ only with $u_C(\hat{\mathbf{x}}/\beta)$.

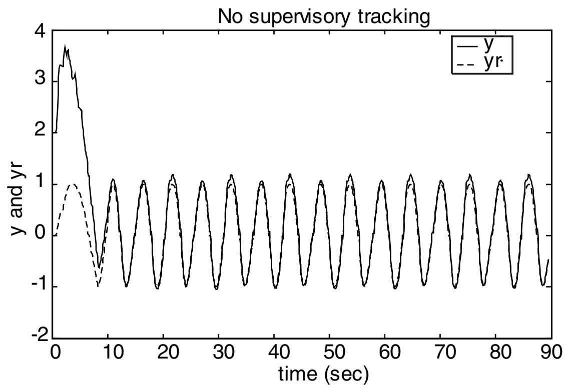

Fig. 7. The trajectory of y and yr only with $u_C(\hat{\mathbf{x}}/\beta)$.

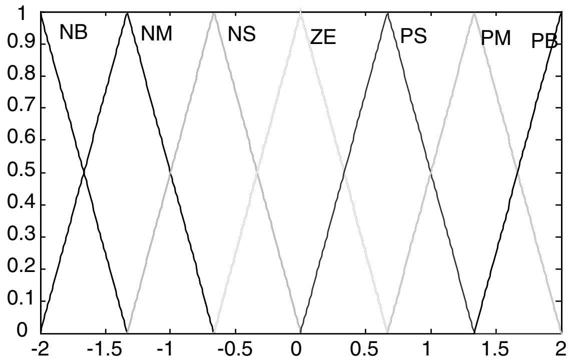

Fig. 8. The membership of e.

Summarizing the above discussion, the design algorithm is described as followings:

Step 1: The observer gain vector is selected as $\mathbf{k}^{\mathrm{T}} = [93 \quad 189]$, the feedback gain vector is chosen as $\mathbf{k}_{\mathrm{c}}^{\mathrm{T}} = [1 \quad 2]$.

Step 2: We select Q in (10b) as $Q = \begin{bmatrix} 11 & 13 \\ 13 & 28 \end{bmatrix}$. By solving (10b), We can obtain the positive definite symmetry matrix $P = \begin{bmatrix} 29 & -14 \\ -14 & 7 \end{bmatrix}$. After simple manipulation, the minimum eigenvalue of matrix Q, $\lambda_{Q\min} = 3.23$.

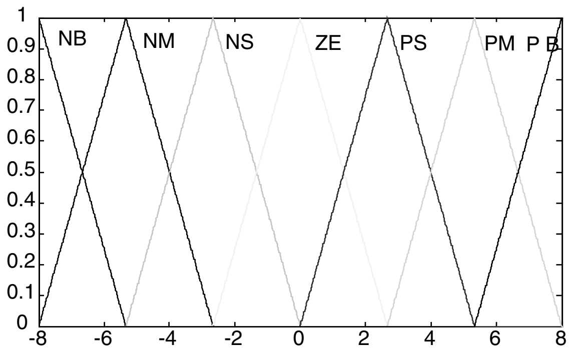

Fig. 9. The membership of $\dot{e}$.

Step 3: The membership functions are selected as follows:

$$
\begin{aligned}
&e_{NB} = \min(1, \max(0, -3e/2 - 2)), && e_{NM} = \max(0, \min(3e/2 + 3, -3e/2 - 1)),\\
&e_{NS} = \max(0, \min(3e/2 + 2, -3e/2)), && e_{ZE} = \max(0, \min(3e/2 + 1, -3e/2 + 1)),\\
&e_{PM} = \max(0, \min(3e/2 - 1, 3e/2 + 3)), && e_{PS} = \max(0, \min(3e/2, -3e/2 + 2)),\\
&e_{PB} = \min(1, \max(0, 3e/2 - 2)), && \dot{e}_{NB} = \min(1, \max(0, -3\dot{e}/8 - 2)),\\
&\dot{e}_{NM} = \max(0, \min(3\dot{e}/8 + 3, -3\dot{e}/8 - 1)), && \dot{e}_{NS} = \max(0, \min(3\dot{e}/8 + 2, -3\dot{e}/8)),\\
&\dot{e}_{ZE} = \max(0, \min(3\dot{e}/8 + 1, -3\dot{e}/8 + 1)), && \dot{e}_{PS} = \max(0, \min(3\dot{e}/8, -3\dot{e}/8 + 2)),\\
&\dot{e}_{PM} = \max(0, \min(3\dot{e}/8, -3\dot{e}/8 + 3)), && \dot{e}_{PB} = \min(1, \max(0, 3\dot{e}/8 - 2)).
\end{aligned}
$$

Step 4: By solving (9), we can obtain $\hat{\mathbf{x}}$. The initial value of state vector is selected as $\mathbf{x}(0) = [2 \quad 2]$.
Step 5: Using (32), (33) to on-line tune parameters α, β.

The trajectories of x_1 and $\hat{x}_1$, x_2 and $\hat{x}_2$ are shown in Figs. 10 and 11, respectively. From these figures, we can see $\hat{x}_1$ and $\hat{x}_2$ can track x_1 and x_2 quickly. The actual output y and the desired output y_r are shown in Fig. 12 in which y can track y_r quickly, moreover the overshoot is smaller than 0.2%. Fig. 13 shows the phase plane of Duffing with $u_C(\hat{\mathbf{x}}/\beta)$ and u_S, u_C.

Example 2. The typical Chua's circuit is shown in Fig. 14, which consists of one linear resistor (R), two capacitors (C_1, C_2), one inductor (L) and a piecewise-linear resistor (g). Chua's has shown to posses very rich nonlinear dynamics such as chaos. Because of its universality, Chua's circuit has attracted much attention and has become a prototype for the investigation of chaos. The dynamic equations of the Chua's chaotic circuit are written as

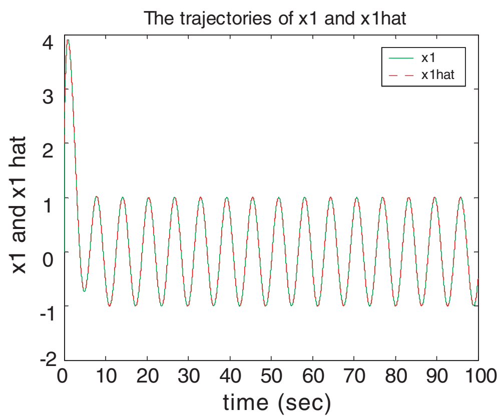

Fig. 10. Trajectory of x_1 and $\hat{x}_1$ with $u_C(\hat{\mathbf{x}}/\beta)$ and u_S, u_C.

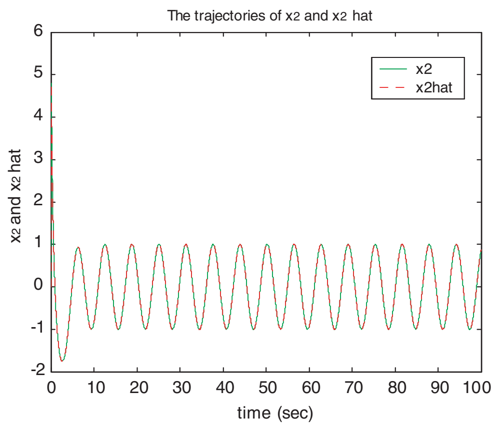

Fig. 11. Trajectory of x_2 and $\hat{x}_2$ with $u_C(\hat{\mathbf{x}}/\beta)$ and u_S, u_C.

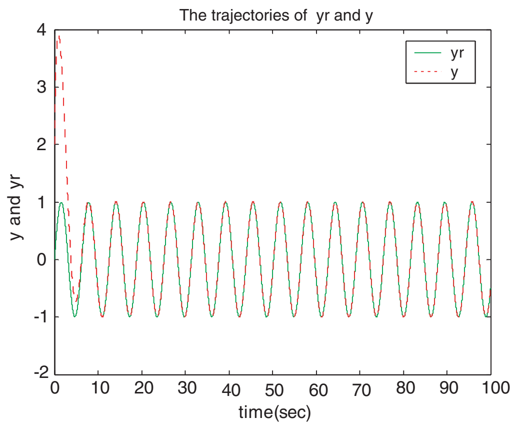

Fig. 12. Trajectory of y and y_r with $u_C(\hat{\mathbf{x}}/\beta)$ and u_S, u_C.

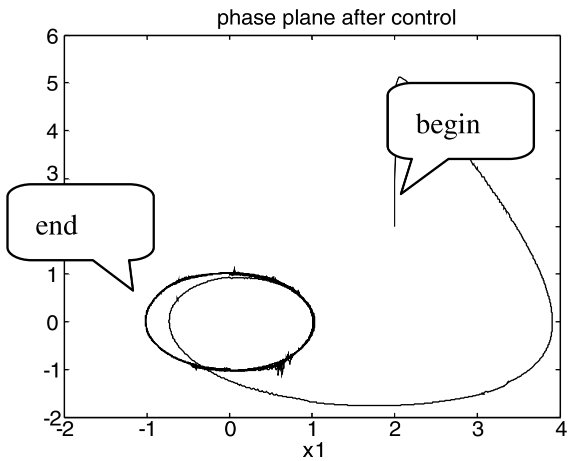

Fig. 13. The phase plane with $u_C(\hat{\mathbf{x}}/\beta)$ and u_S, u_C.

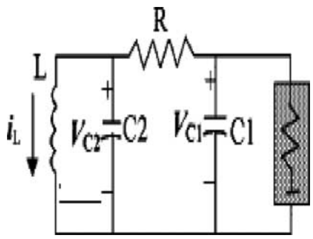

Fig. 14. Chua's chaotic circuit.

$$
\begin{aligned}
\dot{V}_{C1} &= \frac{1}{C1}\left(\frac{1}{R}(V_{C2}-V_{C1})-g(V_{C1})\right),\\
\dot{V}_{C2} &= \frac{1}{C2}\left(\frac{1}{R}(V_{C1}-V_{C2})+i_L\right),\\
\dot{i}_L &= \frac{1}{L}(V_{C1}-R_0 i_L),
\end{aligned}
\tag{43}
$$

where V_{C1}, V_{C2} and i_L are states variables; R_0 is a constant; and g denotes the nonlinear resistor, which is a function of the voltage across the two terminals of C_1. Here we define g as a cubic function as in (44), and its diagram is shown in Fig. 15 [25], where $V_{C1} \in [-d \quad d]$, $d > E > 0$:

$$g(V_{C1}) = aV_{C1} + cV_{C1}^3 \quad (a < 0,\ \ c > 0). \tag{44}$$

From Fig. 25, the bounds for $g(V_{C1})$ are obtained

$$g_1(V_{C1}) = aV_{C1}, \qquad g_2(V_{C1}) = (a + cd^2)V_{C1}. \tag{45}$$

The system (43) can be rewritten as

$$\dot{\mathbf{z}} = G\mathbf{z}(t) + Hg, \tag{46}$$

where $\mathbf{z} = [z_1 \quad z_2 \quad z_3]^T = [V_{C1} \quad V_{C2} \quad i_L]^T$

$$G = \begin{bmatrix} -\frac{1}{C_1R} & \frac{1}{C_1R} & 0 \\ \frac{1}{C_2R} & -\frac{1}{C_2R} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & -\frac{R_0}{L} \end{bmatrix}, \qquad H = \begin{bmatrix} -\frac{1}{C_1} \\ 0 \\ 0 \end{bmatrix}. \tag{47}$$

The obtained state space is not in the standard canonical form defined in (2). Therefore, we need to perform a linear transformation to transform them into the form of (2). Define $\mathbf{z}^*(t) = T^{-1}\mathbf{z}(t)$, where T is a transformation matrix. Using the transformation in [26], the transformed system can be obtained as

$$\dot{\mathbf{z}}^*(t) = T^{-1}GT\mathbf{z}^*(t) + T^{-1}Hg = G^*\mathbf{z}^*(t) + H^*g, \tag{48}$$

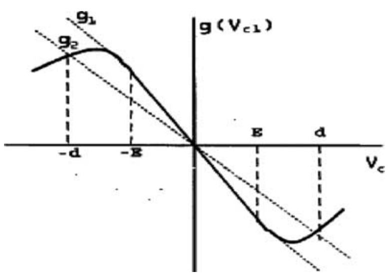

Fig. 15. Nonlinear resistor characteristics.

where $G^* = T^{-1}GT$, $H^* = T^{-1}H$:

$$T = \begin{bmatrix} -\frac{R+R_0}{C_1C_2RL} & -\frac{RR_0C_2+L}{C_1C_2RL} & -\frac{1}{C_1} \\ -\frac{R_0}{C_1C_2RL} & -\frac{1}{C_1C_2R} & 0 \\ \frac{1}{C_1C_2RL} & 0 & 0 \end{bmatrix}, \qquad G^* = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{1}{C_1C_2RL} & -\frac{C_1R+C_2R_0+C_1R_0}{C_1C_2RL} & -\frac{C_1C_2RR_0+C_2L+C_1L}{C_1C_2RL} \end{bmatrix}.$$

Choose the parameters of the Chua's chaotic circuit as following:

$$R = 1.428, \quad R_0 = 0, \quad C_1 = 1, \quad C_2 = 9.5, \quad L = 1.39, \quad a = -0.8, \quad c = 0.044.$$

Therefore, after simple manipulations, we get the transformed system as followings:

$$\dot{z}_1^* = z_2^*, \qquad \dot{z}_2^* = z_3^*, \qquad \dot{z}_3^* = \frac{14}{1485}z_1^* - \frac{168}{9025}z_2^* + \frac{1}{38}z_3^* - \frac{2}{45}\left(\frac{28}{321}z_1^* + \frac{7}{95}z_2^* + z_3^*\right)^3. \tag{49}$$

We will design a variable universe adaptive fuzzy controller with a supervisory controller to force the transformed system to track the given reference signal. For connivance, let $\mathbf{x}$ replace $\mathbf{z}^*$ in (49), therefore, the closed-loop system (49) can be represented as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}(f + gu + d), \tag{50}$$

$$y = [1 \quad 0 \quad 0]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},$$

where $f = \frac{14}{1805}x_1 - \frac{168}{9025}x_2 + \frac{1}{38}x_3 - \frac{2}{45} \times \left(\frac{28}{321}x_1 + \frac{7}{95}x_2 + x_3\right)^3$, $g = 1$, d is the bounded external disturbance. If $u = 0$, then the system (50) is chaotic system, the trajectories of the state variables x_1, x_2, x_3 are shown in Figs. 16–18.

The phase-plane trajectory of x_1x_2 is shown is in Fig. 19. The phase-plane trajectory of x_1x_3 is shown in Fig. 20. The phase-plane trajectory of x_2x_3 is shown in Fig. 21. Fig. 22 shows the space phase-plane trajectory of x_1x_2 x_3.

We design a variable universe adaptive fuzzy controller with a supervisory controller and a robust controller to force the output y of the system to track the given reference signal y_r. In order to satisfy the constrain conditions (16b) in design, we define the following functions:

$$\begin{aligned} |f(\mathbf{x})| &\leqslant \frac{14}{1805} \times |x_1| + \frac{168}{9025} \times |x_2| + \frac{1}{38}|x_3| + \frac{2}{45} \times \left(\frac{28}{321} \times |x_1| + \frac{7}{95} \times |x_2| + |x_3|\right)^3 \\ &\leqslant \frac{14}{1805} \times 50 + \frac{168}{9025} \times 10 + \frac{1}{38} \times 2 + \frac{2}{45} \times \left(\frac{28}{321} \times 50 + \frac{7}{95} \times 10 + 2\right)^3 \\ &\leqslant 13.54 \approx f^{\mathrm{U}}(\mathbf{x}) \approx f^{\mathrm{U}}(\hat{\mathbf{x}}), \end{aligned} \tag{51}$$

$$g^{\mathrm{U}}(\mathbf{x}) \approx g^{\mathrm{U}}(\hat{\mathbf{x}}) = 1.1, \qquad g^{\mathrm{L}}(\mathbf{x}) \approx g^{\mathrm{L}}(\hat{\mathbf{x}}) = 0.9.$$

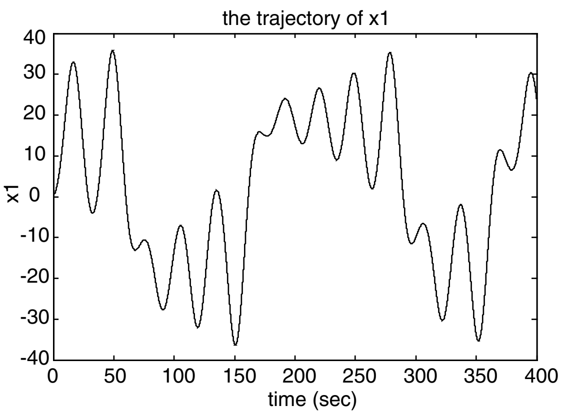

Fig. 16. The x_1 without control.

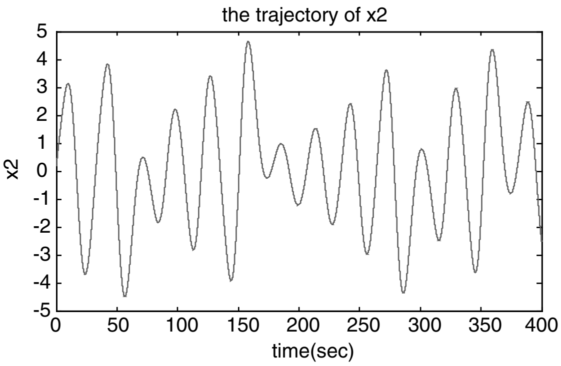

Fig. 17. The x_2 without control.

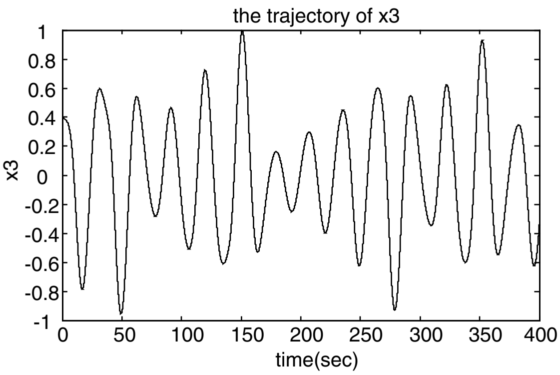

Fig. 18. The x_3 without control.

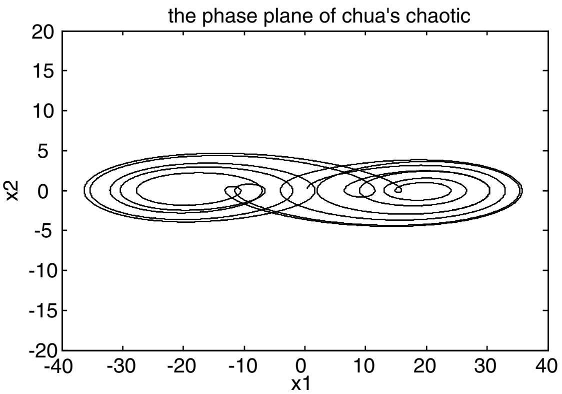

Fig. 19. The phase-plane of x_1 and x_2 without control.

Let the external disturb d is step signal. The membership functions of error and error change are shown in Figs. 8 and 9, respectively.

According to the design procedure, the design is given in the following steps:

Step 1: The observer gain vector is chosen as $\mathbf{k}_c^T = [5 \quad 237 \quad 3]$; the feedback gains vector is selected as $\mathbf{k}_c^T = [12 \quad 13 \quad 3]$. The adaptive coefficient $\gamma = 0.003808$.

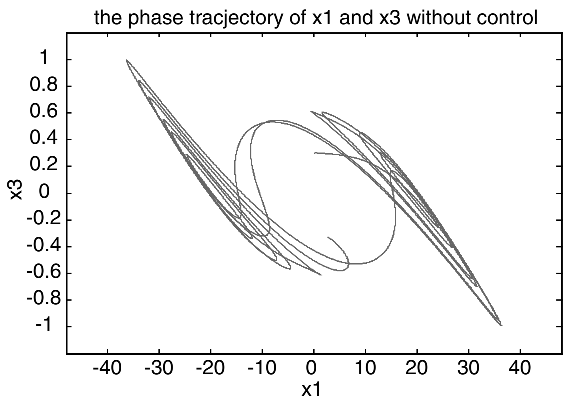

Fig. 20. The phase-plane of x_1x_3 without control.

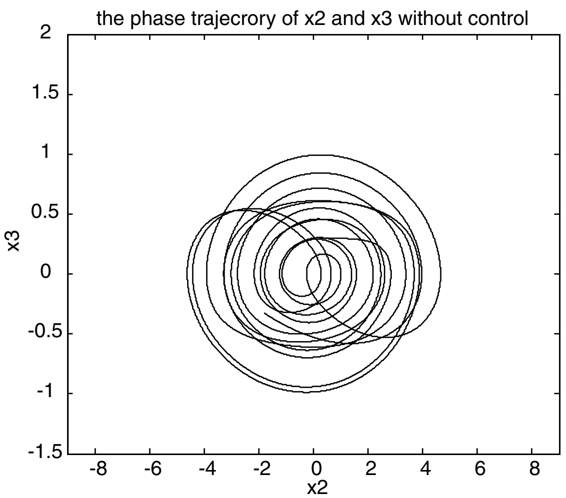

Fig. 21. The phase-plane of x_2x_3 without control.

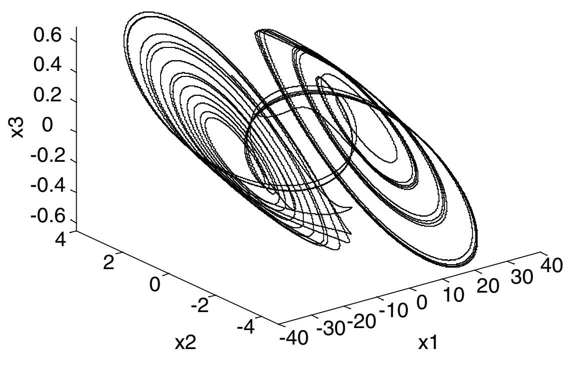

Fig. 22. The space phase-plane of $x_1x_2x_3$ without control.

Step 2: We choose

$$Q=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

in (10b), and then solving (10b), we can obtain the positive definite symmetric matrix

$$P=\begin{bmatrix}143.2233 & -3 & -0.7056\\ -3 & 0.7055 & -3\\ -0.7056 & -3 & 237.1759\end{bmatrix},$$

and the minimum eigenvalue of Q, i.e. $\lambda_{Q\min}$ is 6.

Step 3: The membership functions of e and $\dot{e}$ are selected same as the Duffing system.

Step 4: By solving (9), we can obtain $\hat{\mathbf{x}}$.

Step 5: Use (32), (33) to on-line tune the parameters α and β.

With the proposed control algorithms, the simulation results are shown as following: the trajectories of x_1 and $\hat{x}_1$, x_2 and $\hat{x}_2$ are shown in Figs. 23, 24 respectively. From these figures, we can see that $\hat{x}_1$ and $\hat{x}_2$ can track x_1 and x_2 quickly. The responses of the Chua's chaotic circuit are shown in Figs. 25–27, respectively. The controlled phase-plane trajectory

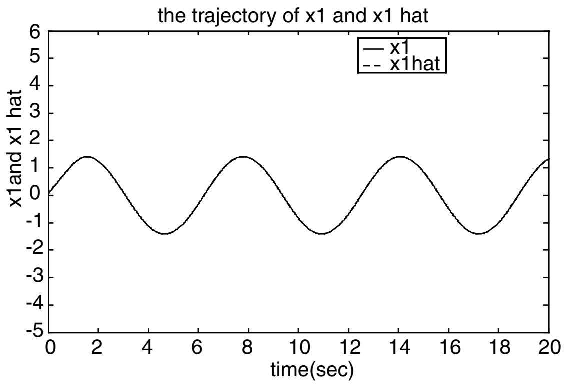

Fig. 23. The controlled x_1 and $\hat{x}_1$.

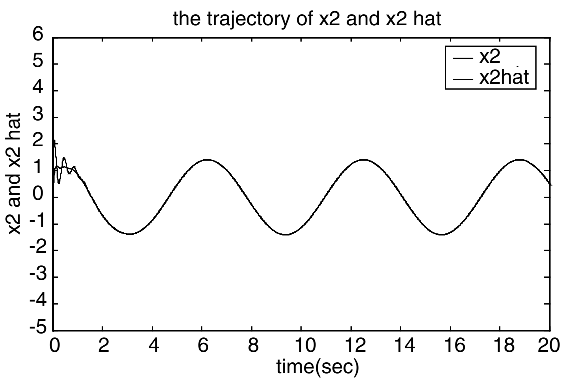

Fig. 24. The controlled x_2 and $\hat{x}_2$.

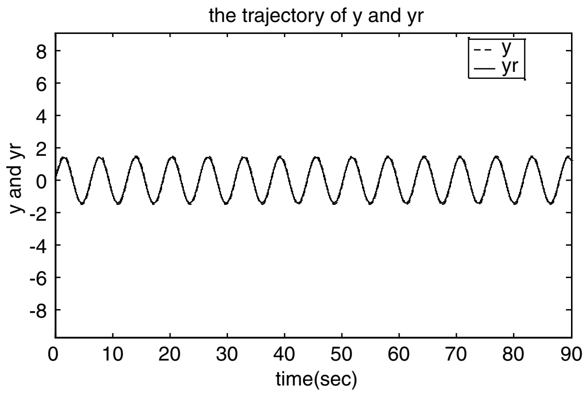

Fig. 25. The controlled y_r and y.

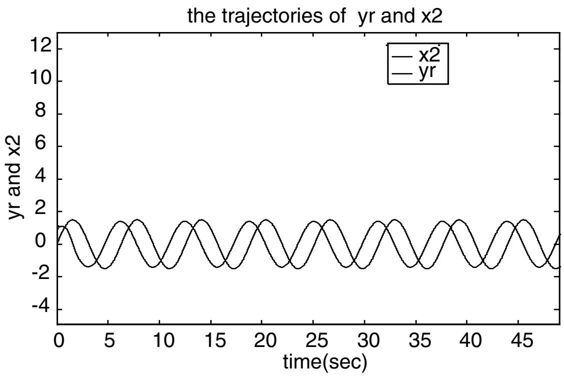

Fig. 26. The controlled of y_r and x_2.

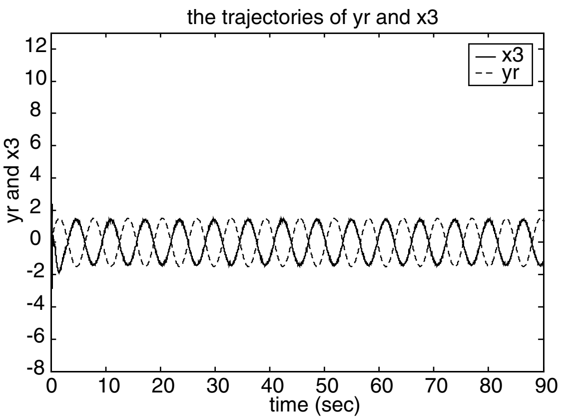

Fig. 27. y_r and x_3 with controller.

of x_1 x_2 is shown in Fig. 28. The controlled space phase-plane trajectory of x_1, x_2, x_3 is shown in Fig. 29, which clearly indicates that the tracking performances are guaranteed by our control algorithms.

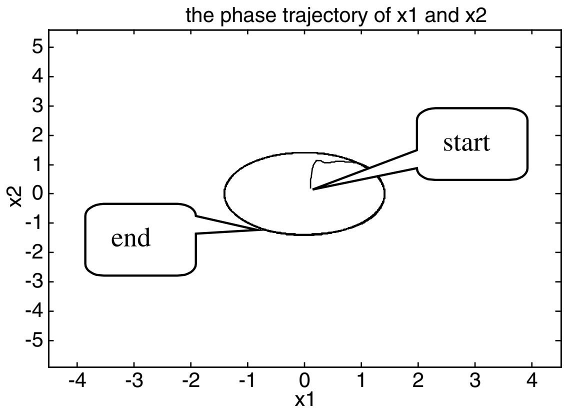

Fig. 28. The phase plane x_1 and x_2 with controller.

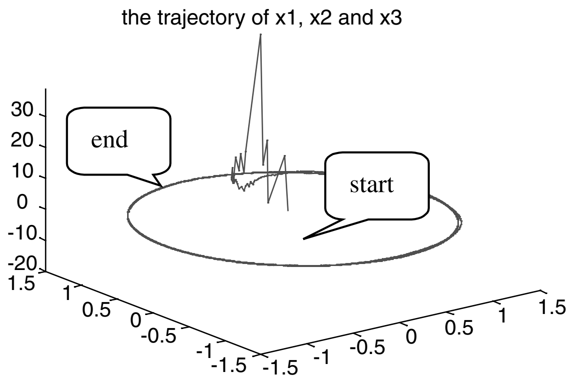

Fig. 29. The phase plane of system with controller.

5. Conclusion

In the paper, a variable universe adaptive fuzzy controller via output-feedback is investigated. The novel types of controllers can on-line tune the contraction and expansion factor. Thus, a number of fuzzy rules are generated. Conventional T–S fuzzy controller must on-line tune all the consequent parameters, which are the centre values of output membership functions. Thus, the realization is difficult. But the variable universe fuzzy controller only on-line tunes the contraction and expansion factor. Thus, the speed of simulation is quick. The supervisory controller is applied to force the states within the compact set. If the variable universe fuzzy controller works well, the supervisory controller is idle; if the system only with variable universe fuzzy controller tends to be unstable, the supervisory controller begins to work in order to guarantee stability. Thus, all the signals involved are bounded. The robust controller is used to attenuate the effect of both approximate error and external disturbance on the tracking error. The proposed control approach is applied to Duffing chaotic system and Chua's chaotic circuit; the simulation results show that the control algorithm is effective.

References

[1] Sastry SS, Isidori A. Adaptive control of linearization systems. IEEE Trans Automat Contr 1989;34:1123–31.
[2] Marino R, Tomei P. Globally adaptive output-feedback control on nonlinear systems, Part I: Linear parameterization. IEEE Trans Automat Contr 1993;38:17–32.
[3] Marino R, Tomei P. Globally adaptive output feedback control on nonlinear systems, Part II: Nonlinear parameterization. IEEE Trans Automat Contr 1993;38:38–48.

[4] Chen BS, Lee CH, Chang YC. H^{∞} tracking design of uncertain nonlinear SISO system: adaptive fuzzy approach. IEEE Trans Fuzzy Syst 1996;4:32–43.
[5] Kang HJ, lee H, Park M. Comments on H^{∞} tracking design of uncertain nonlinear SISO system: adaptive fuzzy approach. IEEE Trans Fuzzy Syst 1998;6:606–6.
[6] Wang L-X. Adaptive fuzzy system and control: design and stability analysis. Englewood Cliffs, NJ: Prentice Hall; 1994.
[7] Wang L-X. Design and analysis of fuzzy identifiers of nonlinear dynamic systems. IEEE Trans Automat Contrl 1995;40:11–23.
[8] Wang L-X, Mendel JM. Fuzzy basis function, universe approximation and orthogonal least-squares learning. IEEE Trans Neural Network 1992;3(5):807–14.
[9] Wang L-X. Stable adaptive control of nonlinear system. IEEE Trans Fuzzy Syst 1993;1:146–55.
[10] Ying H. Sufficient conditions on general fuzzy systems as function approximators. Automatica 1994;30:521–5.
[11] Castro JL. Fuzzy logic controller is universe approximators. IEEE Trans Syst Man Cybernet 1995;25:629–35.
[12] Langari G, Tomizuka M. Stability of fuzzy linguistic control systems. In: Proc 29th IEEE Conf on Decision and Control, 1990. p. 2185–290.
[13] Lee CC. Fuzzy logic in control system: fuzzy logic controller—Part I, Part II. IEEE Trans Syst Man Cybernet 1990;20:404–35.
[14] Khalil HK. Adaptive output-feedback control of nonlinear systems represented by input–output model. IEEE Trans Automat Contr 1996;41:177–88.
[15] Leu YG, Lee TT, Wang WY. Observer-based adaptive fuzzy-neural controller for unknown nonlinear dynamical systems. IEEE Trans Syst Man Cybernet 1999;29:583–91.
[16] Wang J, Rad AB, Chan PT. Indirect adaptive fuzzy sliding model control, Part I: Fuzzy switching. Fuzzy Sets Syst 2001;122:21–30.
[17] Wang CH, Liu HL, Lin TC. Direct adaptive fuzzy-neural control with state-observer and supervisory controller for unknown nonlinear dynamical systems. IEEE Trans Fuzzy Syst 2002;10:39–49.
[18] Moudgal VG, Kwong WA, Passino KM, Yurkovich S. Fuzzy learning control for a flexible-link robot. IEEE Trans Fuzzy Syst 1995;3:199–210.
[19] Takagi T, Sugeno M. Fuzzy identification of systems and its application to modeling and control. IEEE Trans Syst Man Cybernet 1985;15:116–32.
[20] Glower JS, Munighan J. Designing fuzzy controller from a variable structure standpoint. IEEE Trans Fuzzy Syst 1997;5:138–44.
[21] Li H-X. to see the success of fuzzy logic from mathematic essence of fuzzy control. Fuzzy Math 1995;9:1–14. [in Chinese].
[22] Li H-X, Chen CLP. The equivalence between fuzzy logic system and feedfoward neural networks. IEEE Trans Neural Network 2000;11:356–65.
[23] Li H-X, Yen VC. Fuzzy sets and fuzzy decision-making. Boca Rotan, FL: CRC Press; 2000.
[24] Li H-X. Variable universe adaptive fuzzy control on the quadruple inverted pendulum. Sci China, Ser E 2002;45:213–24.
[25] Joo YH, Shieh LS, Chen GR. Hybrid state space fuzzy model based controller with dual-rate sampling for digital control of chaotic systems. IEEE Trans Fuzzy Syst 1999;7:394–408.
[26] Chen CT. Linear system theory and design. third ed. London, UK: Oxford University Press; 1999.