Tracking control based on neural network strategy for robot manipulator

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Abstract

This study presents a sliding-mode neural-network (SMNN) control system for the tracking control of an $n$ rigid-link robot manipulator to achieve high-precision position control. The aim of this study is to overcome some of the shortcomings of conventional robust controllers such as a model-based adaptive controller requires the system dynamics in detail; the fuzzy rule learning scheme has a latent stability problem; an adaptive control scheme for robot manipulator via fuzzy compensator requires strict constrained conditions and prior system knowledge. In the SMNN control system, a neural network controller is developed to mimic an equivalent control law in the sliding-mode control, and a robust controller is designed to curb the system dynamics on the sliding surface for guaranteeing the asymptotic stability property. Moreover, an adaptive bound estimation algorithm is employed to estimate the upper bound of uncertainties. All adaptive learning algorithms in the SMNN control system are derived from the sense of Lyapunov stability analysis, so that system-tracking stability can be guaranteed in the closed-loop system whether the uncertainties occur or not. Computer simulations of a two-link robot manipulator verify the validity of the proposed control strategy in the possible presence of uncertainties and different trajectories. The proposed SMNN control scheme possesses two salient merits: (1) it guarantees the stability of the controlled system, and (2) no constrained conditions and prior knowledge of the controlled plant is required in the design process. This new intelligent methodology provides the designer with an alternative choice to control an $n$ rigid-link robot manipulator.

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1. Introduction

Since the dynamics of robot manipulators are highly nonlinear and may contain uncertain elements such as friction, many efforts have been made in developing control schemes to achieve the precise tracking control of robot manipulators. Conventionally, many control techniques for robot manipulators rely on proportional-integral-derivative (PID)-type controllers in industrial operations due to their simple control structure, ease of design, and low cost [1,2,8]. However, robot manipulators have to face various uncertainties in practical applications, such as payload parameter, internal friction, and external disturbance [11,13,18]. All the uncertain or time-varying factors could affect the system control performance seriously. Many control techniques have been investigated as viable means to improve the shortcomings of the conventional PID-type controllers [4,12,15,20]. Sun and Mills [20] proposed an adaptive-learning control scheme to improve trajectory performance and could guarantee convergence in single and repetitive operational modes. But the control scheme requires the system dynamics in detail. A model-based PID controller was presented by Li et al. [15] to achieve the time-varying tracking control of a robot manipulator. However, it is difficult to establish an appropriate mathematical model for the design of a model-based control system. Thus, the general claim of traditional intelligent control approaches is that they can attenuate the effects of structured parametric uncertainty and unstructured disturbance using their powerful learning ability without prior knowledge of the controlled plant in the design processes.

In the past decade, the applications of intelligent control techniques (fuzzy control or neural-network control) to the motion control for robot manipulators have received considerable attention [5–7,9,10,14,17,22,23]. A control system, which comprises PID control and neural network control, was presented by Chen et al. [5] for improving the control performance of the system in real time. Clifton et al. [6] and Misir et al. [17] designed fuzzy–PID controllers which were applied to the position control of robot manipulators. Huang and Lee [9] suggested a stable self-organizing fuzzy controller for robot motion control. This approach has a learning ability for responding to the time-varying characteristic of a robot manipulator. However, the fuzzy rule learning scheme has a latent stability problem. Yoo and Ham [23] presented two kinds of adaptive control schemes for robot manipulator via fuzzy compensator in order to confront the unpredictable uncertainties. Though the stability of the whole control system can be guaranteed, some strict constrained conditions and prior system knowledge are required in the control process. On the other hand, Kim and Lewis [10] dealt with the application of quadratic optimization for motion control of robotic systems using cerebellar model arithmetic computer neural networks. Lewis et al. [14] developed a multilayer neural-net controller for a general serial-link rigid robot to guarantee the tracking performance. Both system-tracking stability and error convergence can be guaranteed in these neural-based-control systems [10,14]. However, the functional reconstructed error, the neural tuning weights and the high-order term in Taylor series are assumed to be known bounded functions, and some inherent properties of robot manipulator are required in the design process (e.g., skew-symmetry property, bounded system parameters and disturbances). The motivation of this study is to design an intelligent control
scheme for the tracking control of an \( n \)-link robot manipulator to achieve high-precision position control. In the whole design process, no strict constraints and prior knowledge of the controlled plant are required, and the asymptotic stability of the control system can be guaranteed. To accomplish the mentioned motivation, a SMNN control system is developed in this study to control the joint position of an \( n \) rigid-link robot manipulator for periodic motion.

This study is organized as follows. Section 2 presents the dynamic model of an \( n \) rigid-link robot manipulator, and exhibits the design process of a traditional sliding-mode control system. In Section 3, attenuate the effect of uncertainties, a SMNN control system is designed to control an \( n \) rigid-link robot manipulator for periodic motion. The design procedures of the proposed SMNN control system are described in detail. All adaptive learning algorithms in the SMNN control system are derived from the sense of Lyapunov stability analysis, so that system-tracking stability can be guaranteed in the closed-loop system. In Section 4, simulation results of a two-link robot manipulator under possible occurrence of uncertainties are provided to demonstrate the robust control performance of the proposed SMNN control system. Conclusions are drawn in Section 5.

2. Robot dynamics

Consider the dynamic model of an \( n \) rigid-link robot manipulator in the following Lagrange form [21]:

\[
M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) + F(\dot{q}) + T_L(t) = \tau(t),
\]

where \( q, \dot{q}, \ddot{q} \in \mathbb{R}^{n \times 1} \) denote the joint position, velocity, and acceleration vectors, respectively; \( M(q) \in \mathbb{R}^{n \times n} \) is the inertia matrix, which is symmetric and positive definite; \( V_m(q, \dot{q}) \in \mathbb{R}^{n \times n} \) is the centripetal-Coriolis matrix; \( G(q) \in \mathbb{R}^{n \times 1} \) is the gravity vector; \( F(\dot{q}) \in \mathbb{R}^{n \times 1} \) represents the static and dynamic friction terms; \( T_L(t) \in \mathbb{R}^{n \times 1} \) is the vector of disturbances and unmodelled dynamics, and \( \tau(t) \in \mathbb{R}^{n \times 1} \) is the control vector of torque by the joint actuators. The control problem is to find a control law so that the state \( q(t) \) can track the desired command \( q_d(t) \). To achieve this control objective, define the tracking error vector \( e(t) = q_m(t) - q(t) \), in which \( q_m(t) \) represents the reference trajectory. Moreover, define a sliding surface as

\[
S(t) = \left( \frac{d}{dt} + \lambda \right)^2 \int_0^t e(\tau) d\tau,
\]

where \( \lambda \) is a positive constant. Note that, since the function \( S(t) = 0 \) when \( t = 0 \), there is no reaching phase as in the traditional sliding-mode control [3,19]. Differentiating \( S(t) \) with respect to time and using Eq. (1), one can obtain:

\[
\dot{S}(t) = \dot{e}(t) + 2\lambda \ddot{e}(t) + \lambda^2 e(t)
\]

\[
= \ddot{q}_m - M^{-1}(q)[\tau(t) - V_m(q, \dot{q})\dot{q} - G(q) - F(\dot{q}) - T_L(t)]
\]

\[
+ 2\lambda \dot{e}(t) + \lambda^2 e(t)
\]
\[ \dot{q}_m - U(t) + M^{-1}(q)[V_m(q, \dot{q})\dot{q} + G(q) + F(\dot{q}) + T_L(t)] + 2\lambda\dot{e}(t) + \lambda^2 e(t), \]  
\tag{3}

where \( U(t) = M^{-1}(q)\tau(t) \) is viewed as a new control vector. The aforementioned tracking problem is to design a control law \( U(t) \) so that the state remaining on the surface \( S(t) = 0 \) for all \( t > 0 \). In the design of the sliding-mode control system, first the equivalent control law \( U_{eq}(t) \), which will determine the dynamic of the system on the sliding surface, is derived. The equivalent control law is derived by recognizing

\[ \dot{S}(t)|_{U=U_{eq}} = 0. \]  
\tag{4}

Substituting Eq. (4) into Eq. (3), and assuming all system states are available, then

\[ \dot{q}_m - U(t) + M^{-1}(q)[V_m(q, \dot{q})\dot{q} + G(q) + F(\dot{q}) + T_L(t)] + 2\lambda\dot{e}(t) + \lambda^2 e(t) = 0. \]  
\tag{5}

Solving Eq. (5), one can obtain:

\[ U_{eq}(t) = \dot{q}_m + M^{-1}(q)[V_m(q, \dot{q})\dot{q} + G(q) + F(\dot{q}) + T_L(t)] + 2\lambda\dot{e}(t) + \lambda^2 e(t). \]  
\tag{6}

Thus, given \( \dot{S}(t) = 0 \), the dynamics of the system on the sliding surface for \( t \geq 0 \) is given by:

\[ \ddot{e}(t) + 2\lambda\dot{e}(t) + \lambda^2 e(t) = 0. \]  
\tag{7}

Properly choosing the value of \( \lambda \), the desired system dynamics such as rise time, overshoot, and settling time can be easily designed. However, if the system parameters are perturbed or unknown, the equivalent control design can not guarantee the performance specified by Eq. (7). Moreover, the stability of the controlled system may be destroyed. To ensure the system performance designed by Eq. (7) despite the existence of the uncertain system dynamics, a SMNN control system, which comprises a NN controller and a robust controller, is investigated in the following section.

3. Sliding-mode neural-network control system

In order to control the position of an \( n \) rigid-link robot manipulator effectively, a SMNN control system is proposed in this section. The configuration of the proposed SMNN control system is depicted in Fig. 1, in which the reference model is chosen according to the prescribed time-domain control specifications. A general function of a three-layer NN can be represented in the following form [14]:

\[ y = U_{NN}(e, V, W, m, s) \equiv WQ(\dot{e}), \]  
\tag{8}

where the tracking error vector \( e \) is the input state of the NN; \( V \in \mathbb{R}^{k \times n} \) is the input-to-hidden layer interconnection weight matrix, in which \( k \) is the hidden layer
nodes; $W \in \mathbb{R}^{n \times k}$ is the hidden-to-output layer interconnection weight matrix; the active function used in the NN is chosen as $Q(Ve) = \exp[-(Ve - m)^2/s^2] \in \mathbb{R}^{k \times 1}$, in which $m \in \mathbb{R}^{k \times 1}$ and $s \in \mathbb{R}^{k \times 1}$ are the adjustable parameter vectors of the radial basis functions (RBF); $y$ is the output of NN. Thus, an optimal NN controller $U_{\text{NN}}^*$ will be designed to mimic the equivalent control law shown in Eq. (6) such that

$$U_{\text{eq}}(t) = U_{\text{NN}}^*(e, V^*, W^*, m^*, s^*) + \varepsilon \equiv W^*Q^*(V^*e) + \varepsilon,$$

where $\varepsilon$ is a minimum reconstructed error vector; $V^*$, $W^*$, $m^*$ and $s^*$ are optimal parameter vectors of $V$, $W$, $m$ and $s$ in the NN. The control law for the SMNN control system is assumed to take the following form:

$$U(t) = \ddot{U}_{\text{NN}}(e, \dot{V}, \dot{W}, \dot{m}, \dot{s}) + U_s \equiv \dot{W}Q(\dot{V}e) + U_s,$$

where $\ddot{U}_{\text{NN}}$ is a NN controller; $U_s$ is a robust controller; $\dot{V}$, $\dot{W}$, $\dot{m}$ and $\dot{s}$ are some estimates of the optimal parameter vectors, as provided by tuning algorithms to be introduced. The NN control $\ddot{U}_{\text{NN}}$ is used to mimic the equivalent control law due to the uncertain system dynamics, and the robust control $U_s$ is designed to keep the controlled system dynamics on the sliding surface, that is, curb the system dynamics onto $S(t) = 0$ for all times. After some straightforward manipulation, the error equation governing the closed-loop system can be obtained through Eqs. (1), (3) and (6) as

$$\ddot{e}(t) + 2\lambda \dot{e}(t) + \lambda^2 e(t) = U_{\text{eq}}(t) - U(t) = \dot{S}(t).$$

Moreover, $\ddot{U}$ is defined as

$$\ddot{U} = U_{\text{eq}} - U = W^*Q^* + \varepsilon - \dot{W}Q - U_s = \dot{W}Q^* + \dot{W}Q + \varepsilon - U_s,$$
where \( \hat{W} = W^* - \hat{W} \) and \( \hat{Q} = Q^* - \hat{Q} \). The linearization technique is employed to transform the nonlinear active functions into partially linear form so that the expansion of \( \hat{Q} \) in a Taylor series to obtain [14]

\[
\tilde{Q} = \begin{bmatrix}
\tilde{Q}_1 \\
\tilde{Q}_2 \\
\vdots \\
\tilde{Q}_k
\end{bmatrix} = \begin{bmatrix}
\frac{\partial Q_1}{\partial (V e)} \\
\frac{\partial Q_2}{\partial (V e)} \\
\vdots \\
\frac{\partial Q_k}{\partial (V e)}
\end{bmatrix}^T \quad (V^* e - \hat{V} e) + \begin{bmatrix}
\frac{\partial Q_1}{\partial m} \\
\frac{\partial Q_2}{\partial m} \\
\vdots \\
\frac{\partial Q_k}{\partial m}
\end{bmatrix}^T \quad (m^* - \hat{m})
\]

\[
\begin{bmatrix}
\frac{\partial Q_1}{\partial s} \\
\frac{\partial Q_2}{\partial s} \\
\vdots \\
\frac{\partial Q_k}{\partial s}
\end{bmatrix}^T \quad (s^* - \hat{s}) + O_n
\]

or

\[
\tilde{Q} = Q_V \tilde{V} e + Q_m \hat{m} + Q_S \hat{s} + O_n,
\]

where

\[
Q_V = \begin{bmatrix}
\frac{\partial Q_1}{\partial (V e)} \\
\frac{\partial Q_2}{\partial (V e)} \\
\vdots \\
\frac{\partial Q_k}{\partial (V e)}
\end{bmatrix} \in \mathbb{R}^{k \times k},
\]

\[
Q_m = \begin{bmatrix}
\frac{\partial Q_1}{\partial m} \\
\frac{\partial Q_2}{\partial m} \\
\vdots \\
\frac{\partial Q_k}{\partial m}
\end{bmatrix} \in \mathbb{R}^{k \times k},
\]

\[
Q_S = \begin{bmatrix}
\frac{\partial Q_1}{\partial s} \\
\frac{\partial Q_2}{\partial s} \\
\vdots \\
\frac{\partial Q_k}{\partial s}
\end{bmatrix} \in \mathbb{R}^{k \times k},
\]

\[
\tilde{V} = V^* - \hat{V}, \quad \hat{m} = m^* - \hat{m}, \quad \hat{s} = s^* - \hat{s}, \quad O_n \in \mathbb{R}^{k \times 1}
\]

is a vector of higher-order terms. Rewriting Eq. (14), one can obtain:

\[
Q^* = Q + Q_V \tilde{V} e + Q_m \hat{m} + Q_S \hat{s} + O_n.
\]

Substituting Eq. (15) into Eq. (12), it is revealed that

\[
\tilde{U} = W^* Q^* + e - \hat{W} \hat{Q} - U_s = W^* [\hat{Q} + Q_V \tilde{V} e + Q_m \hat{m} + Q_S \hat{s} + O_n] + e - \hat{W} \hat{Q} - U_s
\]
\[
\begin{align*}
    &= (W^* - \dot{W}) \dot{Q} + (W + \dot{W}) Q \dot{V} + (\dot{W} + \ddot{W}) Q_m \dot{m} \\
    &\quad + (\ddot{W} + W) Q S \dot{s} + \varepsilon - U_s + W^* O_n \\
    &= \dot{W} \dot{Q} + \ddot{W} Q \dot{V} + W Q_s \dot{m} + \dot{W} Q_m \dot{m} - U_s + \dot{W} Q_m \dot{m} \\
    &\quad + \dot{W} Q_s \dot{s} + W^* O_n + \varepsilon, \\
\end{align*}
\]

where the uncertain term \( E = \dot{W} Q \dot{V} + W Q_s \dot{m} + \dot{W} Q_m \dot{m} - U_s + \dot{W} Q_m \dot{m} \) is assumed to be bounded by \( \|E\| < \psi \).

**Theorem 1.** Consider the robot dynamic represented by Eq. (1), if the SMNN control law is designed as Eq. (10), in which the adaptation laws of the NN controller are designed as Eqs. (17)–(20) and the robust controller is designed as Eqs. (21) and (22), then the system dynamic can be always kept on the sliding surface such that asymptotical stability can be guaranteed.

\[
\begin{align*}
    \dot{W} &= \eta_1 (\dot{Q} S^T)^T, \\
    \dot{V} &= \eta_2 (e S^T W Q)^T, \\
    \dot{m} &= \eta_3 (S^T \dot{W} Q_m)^T, \\
    \dot{s} &= \eta_4 (S^T \dot{W} Q_s)^T, \\
    U_s &= \hat{\psi}(t) \text{sgn}(S), \\
    \dot{\hat{\psi}}(t) &= \eta_5 S^T \text{sgn}(S),
\end{align*}
\]

where \( \eta_1, \eta_2, \eta_3, \eta_4 \) and \( \eta_5 \) are positive constants; \( \text{sgn}(\cdot) \) is a sign function; \( \hat{\psi} \) is the estimated value of the uncertain term bound \( \psi \).

**Proof.** Define the following Lyapunov function candidate:

\[
\begin{align*}
    L_a(S(t), \hat{\psi}(t), \dot{W}, \dot{V}, \dot{m}, \dot{s}) &= \frac{1}{2} S^T S + \frac{1}{2\eta_1} \text{tr}(\dot{W} \dot{W}^T) + \frac{1}{2\eta_2} \text{tr}(\dot{V} \dot{V}^T) + \frac{1}{2\eta_3} \dot{m}^T \dot{m} \\
    &\quad + \frac{1}{2\eta_4} \dot{s}^T \dot{s} + \frac{1}{2\eta_5} \dot{\psi}^2(t), \\
\end{align*}
\]

where \( \text{tr}(\cdot) \) is the trace operator, and the estimation error is defined as \( \hat{\psi}(t) = \psi - \dot{\hat{\psi}}(t) \). Differentiating Eq. (23), one can obtain that

\[
\begin{align*}
    \dot{L}_a(S(t), \hat{\psi}(t), \dot{W}, \dot{V}, \dot{m}, \dot{s}) &= S^T \dot{S} + \frac{1}{\eta_1} \text{tr}(\dot{W} \dot{W}^T) + \frac{1}{\eta_2} \text{tr}(\dot{V} \dot{V}^T) + \frac{1}{\eta_3} \dot{m}^T \dot{m} + \frac{1}{\eta_4} \dot{s}^T \dot{s} + \frac{1}{\eta_5} \dot{\psi}(t) \dot{\hat{\psi}}(t), \\
\end{align*}
\]

(24)
Substituting Eqs. (11) and (16) into Eq. (24), one can obtain:

\[ \dot{L}_{a} = S^{T}[\hat{W}\dot{Q} + \hat{W}Q_{V}\dot{V} + \hat{W}Q_{m}\dot{m} + \hat{W}Q_{s}\dot{s} - U_{s} + E] - \frac{1}{\eta_{1}} \text{tr}(\hat{W}\hat{W}^{T}) \]

\[ \begin{align*}
&= -\frac{1}{\eta_{2}} \text{tr}(\dot{V}^{T}\dot{V}) - \frac{1}{\eta_{3}} \dot{m}^{T}\dot{m} - \frac{1}{\eta_{4}} \dot{s}^{T}\dot{s} - \frac{1}{\eta_{5}} \dot{\psi}(t)\dot{\psi}(t) \\
&= \text{tr}\left\{ \hat{W}\left[QS^{T} - \frac{1}{\eta_{1}} \dot{W}^{T}\dot{W} \right] \right\} + \text{tr}\left\{ \left[ eS^{T}\hat{W}Q_{V} - \frac{1}{\eta_{2}} \dot{V}^{T}\dot{V} \right] \dot{V} \right\} \\
&\quad + \left[ S^{T}\hat{W}Q_{m} - \frac{1}{\eta_{3}} \dot{m}^{T} \right] \dot{m} + \left[ S^{T}\hat{W}Q_{s} - \frac{1}{\eta_{4}} \dot{s}^{T} \right] \dot{s} \\
&\quad + S^{T}(E - U_{s}) - \frac{1}{\eta_{5}} \dot{\psi}(t)\dot{\psi}(t). \quad (25)
\end{align*} \]

If the adaptation laws of the NN controller are chosen as Eqs. (17)–(20) and the robust controller is designed as Eqs. (21) and (22), Eq. (25) can be rewritten as

\[ \dot{L}_{a} = S^{T}E - \dot{\psi}(t)S^{T}\text{sgn}(S) - \frac{1}{\eta_{5}} \dot{\psi}(t)\dot{\psi}(t) \]

\[ = S^{T}E - \dot{\psi}(t)S^{T}\text{sgn}(S) - \frac{1}{\eta_{5}} [\dot{\psi} - \dot{\psi}(t)]\dot{\psi}(t) \]

\[ = S^{T}E - \dot{\psi}(t)S^{T}\text{sgn}(S) - \frac{1}{\eta_{5}} \dot{\psi}\dot{\psi}(t) + \frac{1}{\eta_{5}} \dot{\psi}(t)\dot{\psi}(t) = S^{T}E - \psi S^{T}\text{sgn}(S) \]

\[ \leq \|S^{T}||E|| - \psi||S^{T}|| = \|S^{T}||(|E|| - \psi) \equiv -z||S^{T}|| \leq 0, \quad (26) \]

where \( z = \psi - \|E|| > 0 \) is a small positive constant. Since \( \dot{L}_{a}(S(t), \dot{\psi}(t), \tilde{W}, \tilde{V}, \tilde{m}, \tilde{s}) \leq 0, \dot{L}_{a}(S(t), \dot{\psi}(t), \tilde{W}, \tilde{V}, \tilde{m}, \tilde{s}) \) is a negative semi-definite function, that is, \( L_{a}(S(t), \dot{\psi}(t), \tilde{W}, \tilde{V}, \tilde{m}, \tilde{s}) \leq L_{a}(S(0), \dot{\psi}(0), \tilde{W}, \tilde{V}, \tilde{m}, \tilde{s}) \), which implies \( S(t), \tilde{W}, \tilde{V}, \tilde{m}, \tilde{s} \) are bounded. Let function \( \Xi(t) \equiv z||S^{T}|| \leq - \dot{L}_{a} \), and integrate function \( \Xi(t) \) with respect to time

\[ \int_{0}^{t} \Xi(\tau) d\tau \leq L_{a}(S(0), \dot{\psi}(0), \tilde{W}, \tilde{V}, \tilde{m}, \tilde{s}) - L_{a}(S(t), \dot{\psi}(t), \tilde{W}, \tilde{V}, \tilde{m}, \tilde{s}). \quad (27) \]

Because \( L_{a}(S(0), \dot{\psi}(0), \tilde{W}, \tilde{V}, \tilde{m}, \tilde{s}) \) is bounded, and \( L_{a}(S(t), \dot{\psi}(t), \tilde{W}, \tilde{V}, \tilde{m}, \tilde{s}) \) is non-increasing and bounded, the following result is obtained:

\[ \lim_{t \to \infty} \int_{0}^{t} \Xi(\tau) d\tau < \infty. \quad (28) \]

Also, \( \dot{\Xi}(t) \) is bounded, so by Barbalat’s Lemma [3,19], it can be shown that \( \lim_{t \to \infty} \Xi(t) = 0 \). That is, \( S(t) \to 0 \) as \( t \to \infty \). As a result, the SMNN control system is asymptotically stable. Moreover, the tracking error vector of the control system, \( e(t) \), will converge to zero according to \( S(t) \to 0 \).  

The adaptive bound estimation algorithm shown in Eq. (22), which has the superiority of simple structure, can guarantee the convergence of tracking error. However,
the guaranteed convergence of tracking error to be zero does not imply convergence of the estimated value of the uncertain term bound to its optimal value. The persistent excitation condition [3,19] should be satisfied for the estimated value to converge to its optimal value. The effectiveness of the proposed SMNN control system can be verified by the following simulation results.

4. Simulation results

For simplicity, a two rigid-link robot manipulator shown in Fig. 2 is utilized in this study to verify the effectiveness of the proposed control scheme. The dynamic model of the adopted robot system can be described in the form of Eq. (1) as [21]

\[
M(q) = \begin{bmatrix}
l_2^2m_2 + l_1^2(m_1 + m_2) + 2l_1l_2m_2 \cos(q_2) & l_2^2m_2 + l_1l_2m_2 \cos(q_2) \\
l_2^2m_2 + l_1l_2m_2 \cos(q_2) & l_2^2m_2
\end{bmatrix},
\]

\[
V_m(q, \dot{q}) = \begin{bmatrix}
-2l_1l_2m_2 \sin(q_2)(\dot{q}_1 \dot{q}_2 + 0.5\dot{q}_2^2)
\end{bmatrix},
\]

\[
G(q) = \begin{bmatrix}
(m_1 + m_2)l_1g \cos(q_1) + l_2m_2 \cos(q_1 + q_2) \\
m_2l_2g \cos(q_1 + q_2)
\end{bmatrix},
\]

where \(q_1\) and \(q_2\) are the angle of joints 1 and 2; \(m_1\) and \(m_2\) are the mass of links 1 and 2; \(l_1\) and \(l_2\) are the length of links 1 and 2; \(g\) is the gravity acceleration. The most important parameters that affect the control performance of the robotic system are the external disturbance \(T_L(t)\), the friction term \(F(\dot{q})\) and the parameter variation of

Fig. 2. Diagram of a two-link robot manipulator.
the mass of link 2, \( m_2 \). Moreover, the system parameters of the two rigid-link robot manipulator are selected as [13]:

\[
l_1 = 1.0 \text{ m}, \quad l_2 = 1.0 \text{ m}, \quad m_1 = 0.8 \text{ kg}, \quad m_2 = 2.3 \text{ kg}, \quad g = 9.8 \text{ m/s}^2. \quad (30)
\]

In addition, the control gains of the SMNN control system are given in the following:

\[
\lambda = 12, \quad \eta_1 = 45, \quad \eta_2 = 45, \quad \eta_3 = 5, \quad \eta_4 = 5, \quad \eta_5 = 48. \quad (31)
\]

All the gains in the SMNN control system are chosen to achieve the best transient control performance in simulation considering the requirement of stability, limitation of control effort and possible operating conditions. The selection of network size is not an easy task and corresponds to the usual model order determination problem. According to the experimental results in [16], the slow convergence of tracking error resulted due to network size being too small. Moreover, if the network size is chosen too large, the improvement of control performance is limited and the computation burden for the CPU is significantly increased. In this study, the NN has two, twenty, and two neurons at the input, hidden, and output layer, respectively, and the random initialization of the network parameters is adopted to initialize the parameters of the network. The effect due to the inaccurate selection of the initialized parameters can be retrieved by the on-line parameter training methodology.

Three conditions, which comprised the nominal condition (\( m_2 = 2.3 \text{ kg} \) and \( T_L = 0 \)) at the beginning, parameter variation condition occurring at 2 s, and the disturbance condition occurring at 6 s are provided to test the robust characteristic of the proposed control system. The parameter variation condition is that 1 kg weight is added to the mass of link 2, i.e., \( m_2 = 3.3 \text{ kg} \); the disturbance condition is that external forces are injected into the robotic system and their shapes are expressed as follows [23]:

\[
T_L(t) = [5 \sin(5t) \quad 5 \sin(5t)]^T. \quad (32)
\]

Moreover, friction forces are also considered in this simulation and are given as [9]

\[
F(q) = [12\dot{q}_1 + 0.5 \text{sgn}(\dot{q}_1) \quad 12\dot{q}_2 + 0.5 \text{sgn}(\dot{q}_2)]^T. \quad (33)
\]

To this end, two simulation cases (Cases 1 and 2) including the aforementioned three conditions are adopted to demonstrate the robust property of the proposed control scheme. Case 1 denotes the three conditions without joint friction, and the friction forces are considered in Case 2. The control objective is to control the joint angle of the two-link robot manipulator to move 1 rad periodically.

In the simulation, the equivalent control law represented in Eq. (6) with nominal system parameters, which is named as a computed torque position control system, is first considered. The computed torque control law is defined as follows:

\[
U(t) = \ddot{q}_m + \dddot{M}^{-1}(q)[\dddot{V}_m(q, \dot{q})\dot{q} + \dddot{G}(q)] + 2\lambda \ddot{e}(t) + \lambda^2 e(t), \quad (34)
\]

where the overbar symbol represents the system parameter in the nominal condition. The block diagram of the computed torque position control system is depicted in Fig. 3. The simulated results of the computed torque position control system due to periodic sinusoidal trajectories at Cases 1 and 2 are depicted in Figs. 4 and 5, respectively. The tracking responses at Case 1 are depicted in Figs. 4(a) and (c); the associated
Fig. 3. Block diagram of computed torque position control system.

tracking errors are depicted in Figs. 4(b) and (d). The tracking responses at Case 2 are depicted in Figs. 5(a) and (c); the associated tracking errors are depicted in Figs. 5(b) and (d). From the simulated results, poor tracking responses after 2 s result due to the occurrence of parameter variations and external disturbance. Though large control gains may solve the problem of delay or degenerate tracking responses, it will result in impractical large control efforts. Therefore, the control gains are difficult to determine due to the unknown uncertainties and unmodelled dynamics, and are ordinarily chosen as a compromise between the stability and control performance.

The proposed SMNN control system shown in Fig. 1 is applied to control the two-link robot manipulator for comparison. The simulated results of the SMNN control system due to periodic sinusoidal trajectories at Cases 1 and 2 are depicted in Figs. 6 and 7. The tracking responses at Case 1 are depicted in Figs. 6(a) and (c); the associated tracking errors are depicted in Figs. 6(b) and (d). The tracking responses at Case 2 are depicted in Figs. 7(a) and (c); the associated tracking errors are depicted in Figs. 7(b) and (d). Since all the parameters and weights of the NN are randomly initialized, the tracking errors are gradually reduced through on-line training methodology of the SMNN control system whether the uncertainties exist or not. Moreover, the robust control performance of the SMNN control system, both in the conditions of joint friction, parameter variation and external disturbance, are obvious as shown in Figs. 6 and 7. Compared with the simulated results of the computed torque position control system, the proposed SMNN control system is effective and yields superior tracking performance. For validating the function of NN controller in the SMNN control system, the simulated results of the NN controller due to periodic sinusoidal trajectories at Cases 1 and 2 are depicted in Figs. 8 and 9. The tracking responses at Case 1 are depicted in Figs. 8(a) and (c); the associated tracking errors are depicted in Figs. 8(b) and (d). The tracking responses at Case 2 are depicted in Figs. 9(a) and (c); the associated tracking errors are depicted in Figs. 9(b) and (d). Compare these results with Figs. 6 and 7, the slow tracking responses result due to the absence of the robust controller. To further test the effectiveness of the proposed control scheme under different reference trajectories, the simulated results of the SMNN control system due to periodic triangular trajectories at Cases 1 and 2 are depicted in Figs. 10 and 11. The tracking responses at Case 1 are depicted in Figs. 10(a) and (c); the associated tracking errors
Fig. 4. Simulated results of computed torque position control system due to periodic sinusoidal trajectory at Case 1: (a) tracking response at joint 1; (b) tracking error at joint 1; (c) tracking response at joint 2; (d) tracking error at joint 2.
Fig. 5. Simulated results of computed torque position control system due to periodic sinusoidal trajectory at Case 2: (a) tracking response at joint 1; (b) tracking error at joint 1; (c) tracking response at joint 2; (d) tracking error at joint 2.
Fig. 6. Simulated results of SMNN control system due to periodic sinusoidal trajectory at Case 1: (a) tracking response at joint 1; (b) tracking error at joint 1; (c) tracking response at joint 2; (d) tracking error at joint 2.
Fig. 7. Simulated results of SMNN control system due to periodic sinusoidal trajectory at Case 2: (a) tracking response at joint 1; (b) tracking error at joint 1; (c) tracking response at joint 2; (d) tracking error at joint 2.
Fig. 8. Simulated results of NN controller due to periodic sinusoidal trajectory at Case 1: (a) tracking response at joint 1; (b) tracking error at joint 1; (c) tracking response at joint 2; (d) tracking error at joint 2.
Fig. 9. Simulated results of NN controller due to periodic sinusoidal trajectory at Case 2: (a) tracking response at joint 1; (b) tracking error at joint 1; (c) tracking response at joint 2; (d) tracking error at joint 2.
Fig. 10. Simulated results of SMNN control system due to periodic triangular trajectory at Case 1: (a) tracking response at joint 1; (b) tracking error at joint 1; (c) tracking response at joint 2; (d) tracking error at joint 2.
Fig. 11. Simulated results of SMNN control system due to periodic triangular trajectory at Case 2: (a) tracking response at joint 1; (b) tracking error at joint 1; (c) tracking response at joint 2; (d) tracking error at joint 2.
are depicted in Figs. 10(b) and (d). The tracking responses at Case 2 are depicted in Figs. 11(a) and (c); the associated tracking errors are depicted in Figs. 11(b) and (d). The tracking errors of the SMNN control system converge quickly, and the robust control characteristics of the proposed position control scheme can be obtained under the occurrence of different reference shape and uncertainties.

5. Conclusions

Since the dynamic characteristics of an $n$ rigid-link robot manipulator are highly nonlinear and coupling, and the complete dynamic model is difficult to obtain precisely, a SMNN control system has been successfully designed in this study to control the joint position of the robot manipulator to achieve high-accuracy position control. In the SMNN control system, all the system dynamics can be unknown and no strict constraints were required in the design process. All adaptive learning algorithms in the SMNN control system were derived in the sense of Lyapunov stability analysis, so that system-tracking stability of the closed-loop system can be guaranteed whether the uncertainties occur or not. To verify the effectiveness of the proposed control scheme, the SMNN control system was implemented to control a two-link robot manipulator. From the simulation results, the joint-position tracking responses can be controlled to closely follow the reference trajectories under a wide range of operating conditions and the occurrence of uncertainties.

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References


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